Fractal Spacetime and Noncommutative Geometry in Quantum and High Energy Physics

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The journal is devoted primarily to the integration of nonlinear dynamics, deterministic chaos and fractals into the very foundation of quantum and high energy physics. In addition we encourage papers dealing with completely novel and partially abstract mathematical theories and techniques in solving fundamental problems in theoretical physics. The journal will accept papers in noncommutative geometry, K-theory, transfinite and intuitionistic set theory, descriptive set theory, number theory, fractal spacetime theories, E-infinity theory, fractal tiling, quantum gravity via fractal Regge calculus, knot theory, wild topology, Euclidean Nash embedding, Lie symmetry groups, fuzzy set theory and fuzzy physics, exotic manifolds in physics, fractal Yang-Mills theories, loop quantum gravity, super string theories, n-categories, E-infinity rings and E-infinity algebra applied to physics, fractal cosmological models, chaotic inflation theories, dark matter and dark energy models, applications of nonlinear dynamics in nano and quantum technologies, advanced experimental techniques in fundamental quantum physics, measurement and quantum paradoxes, fiber bundles and fractal gauge theory, foliation and symplectic geometry in physics. The Editorial Board will intermittingly invite and commission special issues on timely and novel developments in quantum physics and related mathematical subjects.

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Fractal Spacetime and Noncommutative Geometry in Quantum and High Energy Physics

Vol. 3, No.1 2013

Special issue on recent development on dark energy and dark matter
Guest Editor: Ji-Huan He

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On the shoulder of giants
Theoretical physics and cosmology, in fact all branches of science, are deeply indebted to the gigantic effort of measuring the unexpected increased rate of acceleration of the cosmic expansion and the energy density of the entire universe. This work culminated in the more than deserved Nobel Prize in Physics awarded to Prof. S. Perlmutter, Prof. Schmidt and Prof. M. Reiss. With that the missing dark energy is no more a hypothesis. It is no exaggeration to say that dark energy which Mohamed Elnaschie has recently proven it to be the kinetic energy of the quantum wave is signaling a paradigm transition to a new cosmology and new physics. We have never had anything like that in physics or cosmology before. Our guess is that the inclusion of the empty set in E-Infinity theory as in K Theory and in Ultimate L Theory is at the heart of the paradigm shift.

Of course the missing dark energy could have remained a mere missing factor in Einstein's famous equation approximately 1 divided by 22 as already shown by Mohamed Elnaschie. However he could not have ignored the measurement of the unexpected increase in the rate of cosmic expansion. If this increase is real and it is real then dark energy is real as well and gives rise to a form of anti-gravity which is pushing the universe apart. In a sense civilization is intimately linked to the history of cosmological theories and it is what forms our collective Weltanshaung. Mohamed Elnaschie had the great honor and privilege to contribute to this new Weltanshaung in a no-trivial way but he has always maintained that his theory stands on the shoulder of giants. A non-exhaustive picture gallery of it is shown in the next pages. We are dedicating this volume to all those giants and in particular this issue is dedicated to the three Nobel Laureates Perlmutter, Schmidt and Reiss as well as Prof. Mohamed Elnaschie on the occasion of his completing 70 years of active scientific life on the 10th of October 2013.

Ji-Huan He
Soochow University
Welcome address

Ji-Huan He

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Ladies and Gentlemen,

I am immensely pleased and honored to have the chance of hosting you in the Galaxy Hotel, Shanghai, China, and I hope this mini-symposium on dark energy will be live forever in your memory. Developing Shanghai sees developing nonlinear dynamics and developing E-infinity theory as well.

We have successfully organized previous three International Symposium on Nonlinear Dynamics, and this year sees the 4th one, which is held in Suzhou and Shanghai simultaneously. This mini-symposium is organized to celebrate Prof. M.S. El-Naschie's 65 birthday and to announce his great findings on dark energy, which has been an open problem and fascinated many researchers.

2012 ISND is organized as a preeminent event for nonlinear science community and other science communities, such as nanotechnology and high energy physics. It is a great pleasure and honor for Soochow University to host this very important conference.

The goal of this conference is to bring together the researchers from academia and industry to share ideas, problems and developments related to the multifaceted aspects of nonlinear dynamics, thereby capturing both the interest and imagination of the wider communities in various fields, such as mathematics, physics, information science, computational science, biologics, medicine, and others.

In recent years, nonlinear science has had quite a triumph in all conceivable applications in science and technology, especially in high energy physics and nanotechnology. Some of the most fundamental theories can be explained with considerable ease and elegance using nonlinear science.

2012 ISND is unusual, it reveals that nonlinear dynamics works everywhere; This event is special; the mystery of the dark energy is revealed.

One century ago, Albert Einstein combined space and time into his special relativity, and his famous mass-energy equation was obtained;

One century later, Mohamed El-Naschie discovered the transfinite discontinuity of space-time in his E-infinity theory, and the mass-energy equation is completely revised.

Dark energy is a hypothetical form of energy that permeates all of space and tends to accelerate the expansion of the universe. Despite of the great success and undeniable brilliance of the standard model of high-energy physics, it is fair to say that it is by no means perfect. Now E-infinity theory has changed the situation radically. El Naschie actually built a bridge between high-energy physics on the one side and nonlinear dynamics, complex theory, chaos, and fractals on the other, and he benefits tremendously from this scientific cross-fertilization.

According to the E-infinity theory, Prof. El Naschie revealed that dark energy currently accounts for about 95.5% of the total mass–energy of the universe.

Treading the path of El Naschie, we gather together here to share the greatest finding on dark energy since Newton and Einstein, and learn the mystery hidden in the dark energy.

Now let's get together around this new scientific hero, and hear the story of his life recounted and his philosophical enterprise highlighted, seeing El-Naschie’s beautiful poetic spacetime, and knowing how and why space and time should be that way at the quantum gravity resolution.

I anticipate that the mini-Symposium will be a mathematically enriching and socially exciting event and you will be extremely surprised and happy to meet some prominent people from the Middle East and naturally the Far East at this special occasion.

As I said developing Shanghai sees developing nonlinear dynamics and E-infinity theory sees its full blossom in near future in the whole world. This is the subject of the symposium. I can assure you that it will be a scientifically enriching and socially unique day.
Elnaschie standing next to the statue of Einstein and his $E=mc^2$ in Suzhou. The formula has since been revised by dividing it into two parts by Mohamed Elnaschie. The first part is the energy of the quantum particle $E=mc^2/2$. The second part is the energy of the quantum wave $E=mc^2(21/22)$. This is the dark energy which we cannot measure in the usual way because of the wave collapse.
Computing the missing dark energy of a clopen universe which is its own multiverse in addition to being both flat and curved

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Abstract

We approach the problem of the mysteriously missing hypothetical dark energy of the universe from first principles. For that purpose we fuse the large structure as well as the quantum scale of spacetime together and model it using certain Calabi-Yau manifold. That way we derive a rescaled energy-mass special relativity equation which includes quantum effects. The missing energy predicted by this equation is in astonishing, almost perfect agreement with the cosmological measurement of WMAP and supernova analysis. The rationale behind the basic idea of the proposed theory is simply put as follows: Einstein’s special relativity and consequently his equation \( E = mc^2 \) may be considered from the view point of high energy particle physics as a one dimensional theory, i.e. a theory based upon a single elementary particle, the photon. On the other hand the simplest and yet extremely successful standard model of quantum field theory relies on a minimum of 12 particles of which the photon is just one. It is therefore reasonable to assume that the Newtonian expression for kinetic energy \( \frac{1}{2}mv^2 \) could be reduced following a principle of scale relativity by the ratio of one to 12 \(-1 = 11\). This fact also follows from Rayleigh theorem on Eigenvalues which state that restricting the number of generalized coordinates leads to an upper bound rather than a lower bound and lower proper minimization of energy. In other words the maximum quantum relativity approximate magnitude of the energy is \( E \approx (1/2)(1/11)(mc^2) \) when \( v = c \). Compared to the result of the classical special theory of relativity, this is a reduction of nearly 95.45% and matches almost perfectly the cosmic measurements.

The present work claims to give the above qualitative arguments a precise topological foundation using various advanced mathematics and topological theorems. In addition we give a rational ground to the intrinsic dialectic nature of spacetime. In particular we reason that spacetime is both open and closed i.e. clopen in the topological sense as well as being both flat and curved in a way which related the 22 compactified dimensions of bosonic string theory to Einstein’s cosmological constant.

Keywords: Dark Energy, Quantum Gravity, Calabi Yau Manifold, Lorentz Transformation, Dark Extra Dimensions, Yang-Mills Theory.

1. Introduction

From an ardent high energy particle physicist Special relativity and consequently \( E = mc^2 \) where \( E \) is the energy, \( m \) is the mass and \( c \) is the velocity of light is based on a single elementary particle,

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namely the photon [1,2]. By contrast the simplest form of quantum field theory which is referred to
affectionately by physicists as the standard model employs a minimum of 12 elementary particles
which include the photon [1-4]. This number is related to the 12 generators of three combined Lie
symmetry groups of the standard model [4,5]. All the said 12 particles are real and found
experimentally [2,3]. Consequently and again from a particle physicist viewpoint reality is much
richer than a world which is described only by $E = mc^2$ based on a single photon [1]. This way the
scaring discrepancy between theoretical prediction and cosmological measurement as documented by
the hypothetical so called missing 95.5% dark energy of the cosmos [6, 7] could be understood in
principle. Said in different work one could take the view that the said discrepancy is just reflecting the
need for a theory of quantum gravity [8] or at least an improvement of $E = mc^2$ along the lines of
intermediate theories such as the theory of quantum fields in curved spacetime [9]. The present work
will tackle this task starting from very elementary conceptual ideas which we will try to make precise
originally used in high energy physics in the context of superstrings dimensional compactification
[13,14].

2. The basic rationale behind the theory

The main idea of our work is to some extent simple to the point of disbelief. It is the kind of simple
explanation which J.A. Wheeler was hoping for [15]. If $E = mc^2$ is based on 1 photon then “the
improved” $E$ of the quantum plus relativity should at a minimum be based on the 12 elementary
particles of the standard model [1-5]. In an ideal world of Weyl-like gauge theory [1], or what we call
here scale relativity [10], the solution would be simply rescaling

$$E = mc^2$$

using a scaling “exponent” $\lambda$ which is the ratio of one to eleven. That means

$$\lambda = 1/(12 - 1) = 1/11$$

should be used to scale either $E = \frac{1}{2}mv^2$ of Newton where $v \rightarrow c$ or $E = mc^2$ of Einstein but then $\lambda$
will change to $1/(11 + 11^*) = 1/22$ where $11^*$ are eleven super symmetric partners of the 11. In what
follows we will attempt to make this argument watertight, at least within a theory of quantum
relativity rather than quantum gravity where there is a fine distinction between the two [8,9]. This
distinction is however largely irrelevant for computing the hypothetically missing dark energy which
is the main aim of the present short paper [6,7]. Apart of the above, the situation could be considered
from the entirely different view point of variational formulation of an effective Lagrangian. The
number of particles in such a Lagrangian will play the role of generalized coordinates. Restricting the
number so drastically from 12 to only 1, as in special relativity, leads to over estimating the energy
level by Rayleigh’s theorem.

3. The 12 particles of the standard model

The strong, weak and electromagnetic are accounted for in the backbone of quantum field theory [2-4],
i.e. the approximate but brilliant standard model [2] by the combined three symmetry groups SU(3)
SU(2) U(1). As for SU(3), this gives us the 8 massless Gluons, the Messenger particles (i.e. quanta) of
the strong force [2-4]
\[ \text{dim } [\text{SU(3)}] = 3^2 - 1 = 8. \] (1)

Similarly we have \( W^+, W^- \) and the neutral current \( Z^0 \) which mediates the weak force. These three Bosons are given by [2-4]

\[ \text{dim } [\text{SU(2)}] = 2^2 - 1 = 3. \] (2)

Finally \( U(1) \) gives us our familiar photon, the basis of special relativity seen from the view point of particle physics. Thus we have [2-4]

\[ \text{dim } U(1) = 1. \] (3)

All in all we have therefore [2-4]

\[ \text{dim } [\text{SU(3)} \text{ SU(2)} U(1)] = 8 + 3 + 1 = 12. \] (4)

Note that these 12 do not include the Higgs or the until now still experimentally undiscovered graviton explicitly [2-4] although they are included implicitly in the form of a fractal weight. All the same it would be a major step forward if all the 12 standard model particles were involved in \( E = mc^2 \) and not only one photon. In the purely mathematical form of the extremely important Yang-Mills theory [16,17] there are even 3 kinds of photons, two of which are electrically charged. One possible extremely simple idea would be to use Weyl scaling with the scaling exponent \( \lambda = 1/(12 - 1) = 1/11 \) as mentioned earlier on to improve the accuracy of the formula of the single photon special relativity and extend its range of validity. That way we could hopefully explain the missing dark energy. As we will show here, we think we were able to do precisely this to a very large and satisfactory extent. Note also that including the super partners \( 11^* \) brings in gravity indirectly for the same well known reasons of the theory of super gravity [13,14].

4. Calabi-Yau \( \text{cp}^{(5)} \) topology for spacetime

There are five standard Calabi-Yau complete intersections in \( p^{(n)} \). These are \( \text{cp}^{(4)}, \text{cp}^{(5)}, \) two copies, \( \text{cp}^{(6)} \) and \( \text{cp}^{(7)} \). The most important numeric of the holonomy of these manifolds are the Betti numbers [13,14]. Using the Hodge diamond and standard notation we have \( h^{1,1} = 1 \) for all the five while for \( h^{2,1} \) we have 101, 89, 73, 73 and 65 respectively. In particular \( h^{2,1} = 101 \) and \( h^{2,1} = 89 \) may be interpreted as the dimensions of a modular space [11-14]. Now the spacetime dimension of special relativity is simply 4 and if M-theory and Heterotic string theory is correct, then quantum gravity “spacetime” looks more like a \( \text{cp}^{(6)} \), manifold than the smooth 4 dimensional space of special and even general relativity. Assuming again the sway of Weyl original gauge theory, then we could expect that \( E \) could be scaled as in Nottale’s scale relativity [10] by the ratio of 4-D dimension of relativity to the 89 dimensions of \( h^{2,1} \) of \( \text{cp}^{(5)} \) [11].

That means

\[ \lambda = \frac{D^{(4)}}{h^{2,1}} = \frac{4}{89} = \frac{1}{22.5}. \] (5)
It is possible to show that this result is basically identical to our previous one $\lambda = 1/11$ and that $22 = (2)(11)$ where the eleven are nothing else but the dimensions of Witten’s eleven dimensional M-theory [17]. The exact transfinite expression is in fact

$$\lambda = \frac{1}{22 + k}$$

(6)

where $k = \phi^3(1 - \phi^3) = 0.18033989$, $\phi = (\sqrt{5} - 1) / 2$ but this will not be discussed here as it is not our main concern at this point.

5. Quantum relativity and Weyl scaling

From the preceding analysis we could rederive $E = mc^2$ from first principle and allow in that derivation for an Immirzi-like parameter [18] similar to that deduced in the theory of loop quantum gravity [1,18]. Such analysis is possible and leads to a factor $Y \simeq 1/22$, almost the same result obtained in the previous section. However this all shows that there is a deep simplicity hidden here in the form of scale relativity or Weyl scaling [10,19]. The principle is explicitly included in Nottale’s theory [10] and implicitly in Magueijo-Smolin theory of varying speeds of light and Planck energy invariance [19]. Thus from the Newtonian kinetic energy

$$E = \frac{1}{2}mv^2.$$  

(7)

We could introduce the following scaled energy

$$E \rightarrow E_{QR}$$

(8)

when

$$\frac{1}{2} \rightarrow \frac{1}{22}$$

(9)

and

$$v \rightarrow c$$

(10)

so that we find the energy from a quantum relativity formula

$$E_{QR} = \frac{mc^2}{22}.$$  

(11)

The 22 could also be naively but correctly interpreted as the “dark” Bosonic dimensions of the 26 dimensional string theory $26 - 4 = 22$ where the 4 are the dimensions of ordinary spacetime. Compared to special relativity we now have a reduction in $E$ by $[1 - (1/22)] (100) = 95.4545\%$ in almost full agreement with cosmic measurements [6, 7]. Seen that way our spacetime must be regarded as being both flat and curved. The flat parts are the 4 large ordinary spacetime dimensions. The geometrically intrinsically curved part are the 22 extra dimensions causing the 1/22 reduction factor of $E$ and could be consequently interpreted as akin to Einstein’s cosmological constant being
the driving force behind the relatively recent observed increase in the acceleration of the expansion of the Universe [1, 6, 7, 19], [23,24].

6. Missing dark energy for $\text{cp}^{(4)}$ Calabi-Yau manifold and eleven dimensional super gravity

Next we look at a more sophisticated derivation of the quantum Lorentz factor $\gamma_0$ or the scale relativity [10,19] “exponent” which we found approximately to be $\lambda = 1/22$. We recall that $b_2 = h^{2,1} = 1$ for the space of special relativity [10]. In the eleven dimensions of super gravity this could be boosted to $(11) (b_2) = 11$. On the other hand the holographic boundary given by $\text{SL}(2,7)$ Lie symmetry group [5,20] has exactly 336 degrees of freedom [1,20] representing the entire force of the strong interaction. When embedded in its intrinsic space $D = 7$ then we find $336 + 7 = 343$ degrees of freedom or isometries representing particle-like objects at ultra high energy [1,20]. It is important to note the close connection here to automorphism of the group made of summing over all the two and three Stein spaces [21]. There are only 17 such spaces and the sum is given exactly by [21]

$$\sum_{1}^{17} |\text{Stein}| = 686. \quad (12)$$

Thus we have

$$\text{Dim } [\text{SL}(2,7) + 7] = \frac{1}{2} \sum_{1}^{17} |\text{Stein}|$$
$$= \left(\frac{1}{2}\right) [(137)(5) + 1]$$
$$= \frac{1}{2} (686)$$
$$= 343. \quad (13)$$

Now the dimension of $\text{cp}^{(4)}$, namely $h^{2,1} = 101$ [11] is already included in deriving $(h^{1,1})(11) = 11$ of the eleven dimensional M-theory [22]. Consequently to obtain the correct scaling exponent for $E$ we must subtract 101 from 343 and find 242 dimensions. Therefore our scaling exponent can be written and calculated readily as

$$\lambda = \frac{\text{D}^4}{\text{SL}(2,7) + 7 - \text{dim cp}^{(4)}}$$
$$= \frac{11}{(336 + 7) - 101}$$
$$= \frac{242}{242}$$
$$= \frac{1}{22} \quad (14)$$

in complete agreement with what we found earlier on.
7. **The justification for M-theory’s eleven dimensions and the relation to Yang-Mills theory and \( \lambda = 1/22 \) scaling**

Witten gives a simple convincing argument for why 11 dimensions [22]. In short the standard model without gravity or Higgs needs seven (7) dimensions to accommodate SU(3) SU(2) U(1) [2,3,20]. This added to 4 spacetime of relativity gives us 7 + 4 = 11 dimensions of M-theory. Here we could look at the situation in a slightly different way leading to the following interpretation. Yang-Mills in field theoretical terms is a 6 dimensional theory [16,17] because it has 3 electrical fields and 3 magnetic fields [16,17]. On the other hand general relativity has the familiar four dimensions. To unify Yang-Mills with general relativity we need one extra dimension like in all the Kaluza-Klein theories. Thus we have

\[
D = D(\text{Yang-Mills}) + D(\text{unification}) + D(\text{general relativity}) \\
= D(\text{Yang-Mills}) + 1 + 4 \\
= D(\text{Yang-Mills}) + D(\text{Kaluza-Klein}) \\
= 6 + 5 \\
= 11(\text{Witten M-theory}).
\]  

(15)

If we have one more time dimension then one finds \( D = 12 \) of C. Vafa [1, 22]. It is interesting then to see how our \( \lambda = 1/22 \) could have so many related but different interpretations. For instance the 3 different photons of Yang-Mills theory [16, 17] could be added to 20 degrees of freedom of Einstein’s gravity then we subtract the familiar light photon and obtain \( 3 + 20 - 1 = 22 \). These 22 particle-like isometries could be understood as 11 dimensions added to 11 dimensions to find 22 super symmetric dimensions if one wants to see it that way [1,2,13]. The result is a scale relativity exponent equal to the ratio of a single photon representing special relativity to 22 elementary massless gauge Bosons [1-3] which were never included in any way in the original derivation of \( E = mc^2 \) of special relativity [1]. The reader should note that a naive scaling does in fact exist in a self-similarity between Newton’s \( E = mc^2 / 2 \) and Einstein’s \( E = mc^2 \) so that our result for quantum relativity \( E = mc^2 / 22 \) is on this level no surprise and completely in keeping with this simple principle of fundamental self-similarity. In short ordinary energy \( mc^2 / 22 \) is the energy of the quantum particle which dark energy. \( mc^2(21/22) \) is the energy of the quantum wave. The sum is equal to Einstein energy \( mc^2 \).

8. **The generalized Lorentz transformation**

We could not conclude the present discussion without at least sketching how \( \lambda \) could be arrived at without scale relativity [10] via the familiar Lorentz symmetry invariance of the Lorentzian group L(4) [20]. The situation here is analogous to a more studied case relevant to quantum gravity [8]. More precisely it arises in connection with loop quantum gravity that we look at three Lorentzian formalism [18]. The first may be called Euclidean connection with the by now familiar imaginary time and Wick rotation [18]

\[
A^i = w^i + w^{oi} \tag{16}
\]

where \( i = \sqrt{-1} \). The second possibility is defined as above but \( A \) transforms as a connection under SO(3) and therefore it is a real one. The third possibility which for us here is the most important is to use a Lorentzian connection [18].
where $Y$ turned out via comparison with some results of black hole thermodynamics to be given by $\log 2 / (\pi^2/3)$. This is the famous Ashtkar-Sen-Barbero-Immirzi parameter [18] without which no result in loop quantum gravity will be numerically correct. Working so hard on mathematically very difficult problems one could sometimes forget that physics of reality is about numbers [1]. For sure the calamity of the missing dark energy in the universe is about a number which was found to be $4.5\%$ while we were expecting 100% [6].

Following the same procedure outlined above and accepting complex value for the Lorentzian boost, then taking the modulus, the end result is an energy-mass equation identical to the equation of special relativity with the single difference that there is a parameter $Y$ which is not unity and which gives $E_{\text{max}}$ when it is

$$Y = \gamma = \frac{1}{22.18033989} = \lambda$$

from which one finds

$$E_{Q,R} = \frac{mc^2}{22.18033989}.$$  

Although this is an exact solution, as far as the authors are concerned, they feel that the much simpler and more intuitive arguments used in the earlier sections may be more convincing.

**9. Conclusion**

The energy-mass equation of special relativity cannot give even a second rate approximation to reality when applied to the entire cosmos [6,7]. By Rayleigh’s theorem on Eigenvalues, the drastic reduction of the number of generalized coordinates to only one photon will clearly lead to an excessive upper bound for the energy. Nevertheless a Weyl – Nottale scaling argument leads in a very simple direct way to a reduction factor of $E = mc^2$ to become $E_{Q,R} = \gamma mc^2$ where for $E_{Q,R}$ (max) the factor gamma is given by a scaling exponent equal to $\lambda = 1/22$. This scaling exponent may be obtained mathematically in a variety of ways. Here we used two standard forms of the intrinsically 6 dimensional Calabi-Yau manifold used in superstring theories for compactification [13,14]. For instance knowing that the Betti number of spacetime of special relativity is equal to unity while the same number for Calabi-Yau $\text{cp}^{(5)}$ is 89, we can easily show that $\lambda$ is given approximately $4/89 = 22.5$ where 4 is the dimensionality of Einstein’s spacetime and 89 is the dimensionality of modular manifold corresponding to $\text{cp}^{(5)}$ which models in this case the rugged topology of real spacetime at very large cosmic distances as it previously solved the problem at extremely small quantum scales in string theory [12-14]. In conclusion we note following previous result by the first author that $\lambda = 1/22$ implies fractal spacetime with ground states made of empty sets. Consequently, our spacetime is clopen in the topological sense. In addition it is flat and curved and not only one or the other being the driving force behind the observed increased expansion of the Universe where $(1 - (1/22)) = 21/22$ of the curved hidden dark dimensions play the role of Einstein’s Cosmological Constant [23,24].
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Dark energy of the quantum Hawking-Hartle wave of the cosmos from the holographic boundary and Lie symmetry groups – Exact computation and physical interpretation

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Abstract

Utilizing scale relativistic invariance we show how Einstein’s mass-energy relations of special relativity can easily be converted into a quantum relativity equation giving accurate predictions for the missing dark energy of the cosmos. Instead of working with the bulk for instance E8E8 exceptional Lie symmetry groups, we start from the holographic boundary in its simplest form given by the original Klein modular curve of SL(2,7) simple linear Lie group. The agreement is almost surreal between our results i.e. 95.4915028% of the theoretical energy of the universe is dark energy and those found from the exacting WMAP measurement as well as the analysis of the supernova.

Keywords: Dark energy, Exceptional Lie symmetry groups, Holographic principles, Quantum set theory and groups, Penrose tiling.

1. Introduction

The relatively recent history of cutting edge research in theoretical physics has made it commonplace to say that asking the right questions is nearly half of the right answer. It is a fact that we are still asking a few misleading questions such as whether the Universe is open or closed, and whether it is flat or even in the absence of matter still preserves some intrinsic curvature. However, on deep reflection we must conclude, based on the topology of Cantor sets, that the universe is both open and closed i.e. clopen [36] as noted some time ago by El Naschie. Similarly it is flat and curved at the same time. Seen as a mere 3 + 1 dimensional space, it is flat. However seen as a string space with 22 extra dimensions, it is a highly curved space. We are all subject to Gödel’s theorem and we could never be completely free from prejudice and wrongly posed questions. Nevertheless, we will try to free ourselves in the present work from at least a few well-established wrong conclusions, for instance that quantum entanglement could not affect Einstein’s special relativity, the famous formula relating energy to mass and the speed of light [37-40].

The postulate of the constancy of the speed of light served modern theoretical physics extremely well in theory and experiment [1], that is, until we started enquiring about physics at ultra high energy and moving outwardly to intergalactic scales as well as inwardly towards quantum gravity and Planck

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scales [1-14]. In hindsight it was preprogrammed that we will reach the limit of the predictability power of what we thought of for quite a long time to be the unshakable pillar of theoretical physics, i.e. Einstein’s energy-mass equation of special relativity $E = mc^2$ where $E$ is the energy, $m$ is the mass and $c$ is the speed of light. The deviation between the total energy in the cosmos according to this equation from accurate measurements by COBE, WMAP [15] and the analysis of a relatively recent supernova type Ia (SNIa) by the 2011 Nobel Laureates in Physics, Reiss et al. and Perlmutter et al. [16] is not simply ten or twenty percent. The discrepancy between theory and measurement is really horrendous and amounts to 22 times less energy than predicted theoretically [15]. El Naschie suspected for quite a long time that this discrepancy must be due to the empty voids in the fabric of spacetime itself [3,4, 11-14]. Consequently it must be possible to resolve the discrepancy by taking the fractal nature of the geometry and topology of quantum micro-spacetime into consideration when deriving the energy mass relationship. In other words, a theory of quantum gravity or quantum relativity needs to be developed first. Luckily one can shortcut this road and fuse quantum mechanics with relativity in a rather direct way. The main idea stems from the realization that fractals formally have infinitely many dimensions [6,11] and that may indirectly explain the relative success of theories with extra hidden dimensions such as Heterotic strings, super gravity and the eleven dimensional M-theory of Witten [17-19].

In superstrings one starts with a sufficiently large Lie symmetry group which turns out to be either $\text{E}_8\text{E}_8$ or $\text{SO}(32)$, both with 496 dimensions or generators [20-23]. For the simple standard model at the relatively modest energy scale of the electroweak, quite some time ago it was found that an optimal minimum of 12 massless gauge bosons are sufficient [19-26]. However as we stated, probing higher energy scales and enquiring about not only electroweak unification but far more ambitious unification such as $\text{SU}(5)$ and $\text{SO}(10)$ grand unification [19-23] and more than that, attempting a theory of everything involving gravity, scientists were forced to use 496 gauge bosons to replace the modest 12 of the standard model [19-23]. In view of this high dimensionality an idea started pressing itself: if these various invisible dimensions could be physical, real and not only a clever mathematical tool, then there is a possibility that these extra dark dimensions are behind the dark energy and may give a geometrical-topological explanation to Einstein’s cosmological constant. We looked at this possibility using the bulk symmetry group $|\text{E}_8\text{E}_8| = 496$. However, the theory of the holographic boundary must reach the same conclusion and may provide even greater insight into the physical mathematical meaning of dark energy. The reason for this is quite straight forward and will be discussed in the next section. Suffice here for the moment to mention that the result of the holographic boundary analysis [24-26] turns out as expected to be identical to that obtained using the bulk; and that the modified energy-mass relationship of quantum relativity at the holographic boundary predicts that dark energy amounts to 95.49% of the total energy of the universe in full agreement with WMAP measurement [15] and the supernova analysis [15,16].

2. The holographic boundary and Lorentzian invariance

The holographic boundary [27] in its most illuminating form may be taken as the surface of a bulk given by $|\text{E}_8\text{E}_8| = 496$ exceptional Lie symmetry group [25]. This surface or hologram is represented by the classical Klein modular group $\Gamma(7)$ [14,20-22] with its 336 degrees of freedom being the number of the familiar triangles tiling a 2D flat space. The symmetry group of this modular curve is the $\text{SL}(2,R)$ where $R = n = 7$ expected dimensions of $\Gamma(7)$ [11,20,21]

$$
\dim \text{SL}(2,n) = |\text{SL}(2,n)| = n(n^2 - 1) = 7(7^2 - 1) = 336.
$$
The fundamental equation of the conservation of dimensions thus reads \[ [E_8E_8] = [SL(2,7)] + [SU(2)] + [E_6E_6] + [U(1)] \]
\[ = 336 + 3 + 2(78) + 1 \]
\[ = 336 + 3 + 156 + 1 \]

where \( E_6 \) is an exceptional Lie symmetry group of dimension \( |E_6| = 78 \), \( |SU(2)| = 2^2 - 1 = 3 \) gives us the weak particles \( W^+, W^- \) and \( Z^0 \) while \( |U(1)| = 1 \) represents an ordinary photon \[20-23\]. We can see clearly that \[20,23\]
\[ [E_6E_6] + [U(1)] = 157 \]
gives us the pure gravity degrees of freedom for \( d = 8 \), namely
\[ G_p = d(d - 3)/2 \]
\[ = (8)(8 - 3)/2 \]
\[ = (5)(8)/2 \]
\[ = 20. \]

This is the same as the independent components of the Riemannian curvature in 4 dimensions \[20, 21\]
\[ R^{(4)} = n^2(n^2 - 1)/12 \]
\[ = 4^2(4^2 - 1)/12 \]
\[ = 20. \]

In addition the 157 includes the degrees of freedom of the electromagnetic manifold \( \alpha_\circ = 137 \) degrees of freedom being the integer value of the inverse of the Sommerfield fine structure constant at ordinary low energy, i.e. energy of the electron mass. In other words we have \[20-22\]
\[ [E_6] + [E_6] + [U(1)] = \alpha_\circ + G_p \]
\[ = 137 + 20 \]
\[ = 157. \]

The bulk is thus represented dimensionally via the dimension of 4 fundamental groups of the four fundamental forces \[20-22\]

(a) \( |SL(2,7)| = 336 \) for the strong force
(b) \( |SU(2)| = 3 \) for the weak force
(c) \( \alpha_\circ = 137 \) for electromagnetism
(d) \( G_p^{(8)} = R^{(4)} = 20 \) for Einstein gravity.

Having come that far it must be said that \( |E_8E_8| = 496 \) and
\[ SL(2,7) + SU(3) = 336 + 3 \]
\[ = 339 \]
are only the integer approximations to an exact transfinite value [20-22]. The same applies to $\alpha$, which is [20,22]
\[
\alpha = 137 + k = 137.082039325.
\]

Similarly [25]
\[
|E_8|_c = 496 - k^2 = 496 - (0.18033989)^2 = 495.9674775
\]
and [25,26]
\[
|SL(2,7)|_c = |SL(2,7) + SU(3)|_c = 336 + 16k = 338.8854382.
\]

However, $R^{(4)} = G^{(5)}$ remains sharp as they are equal to 20. The exact balance is thus [19-23]
\[
|E_8|_c = |SL(2,7)|_c + \alpha + R^{(4)} = 338.8854382 + 137.0820398 + 20 = 496 - k^2 = 495.9674775,
\]

3. Preservation of Lorentz invariance

The important picture which arises from the preceding analysis is that the holographic boundary is given exactly by a formally infinite dimensional group with graded infinite dimensions [1, 30], but the behavior is nevertheless bounded by a horizon at infinity like an Escher drawing of the tiling of angels and devils [30]. This 336 + 16k is a weighted dimension of infinite group tiling on the 2D plane from the center to infinity. Yet it is bounded within a circle whose border we could never reach although it has a so called isomorphic length after which the quasi-periodic tiling of $|SL(2,7)|_c$ reproduces itself. This gives one the feeling that if eyes were closed, one had never moved from the center [14]. The maximum isometric length to experience this phenomenon, as is well known, is equal to [14]
\[
\ell = [\frac{4 + \phi^3}{2}]\rho.
\]

Realizing that $|SL(2,7)|_c = 336 + 16k$ is identical to the group $SL(2,C)$, i.e. the complex version of $SL(2,R)$ [20-23], and that $|SL(2,7)|_c$ is the transfinite version of a bounded and finitely made infinite $SL(2,C)$, we see that we replaced complex numbers with a quasi-periodic self affine fractal-like compactified Klein modular curve [21-23]. In turn we know that the Lorentzian group $L(4)$ is a subgroup of the Poincaré group. Consequently our $|SL(2,7)|_c = 336 + 16k$ can be shown to preserve Lorentzian invariance [23] without adhering to the consistency of the speed of light. Put that way,
there is no limit on energy while preserving $1/L_p$ of Planck as invariant replacing the velocity of light where $L_p$ is the Planck length as in the VSL theory of Magueijo and Smolin [31]. The same thing applies to the scale relativity theory of Nottale [4]. We note in anticipation of our next discussion that the modification of Einstein’s $E = mc^2$ takes in the case of the holographic boundary of the form [27,30]

$$E_{QG} = \gamma_H mc^2$$

where $\gamma_H$ is given by

$$\gamma_H = \frac{R^{(4)} - D^{(4)}}{[SL(2,7)] + (R^{(4)} - D^{(4)})} = \frac{20 - 4}{(336 + 16k) + 16} = \frac{16}{354.8854383} = 0.04508497187 = 1/(22 + k)$$

where $k = \phi^i (\phi^i - 1) = 0.180339325$, and $\phi = (\sqrt{5} - 1)/2$. This is nearly the 22-fold error in estimating the dark energy which we mentioned earlier. Now we regress to give in detail a simplified integer approximation of the factor $\gamma = 1/22 + k = 1/22.18033989$. A comparison between the bulk and the holographic boundary analysis is given again for clarity in integer approximation in Table 1. In section 5 we will return to the question of reproducing the quasi-periodic tiling of $SL(2,C)$ and its relation to the fundamental question of an open or closed universe or even multiverse.

4. Simple integer approximation for determining the quantum relativity Lorentz factor $\gamma_H$

The factor by which the simple Einstein equation should be multiplied to scale it to the right energy scale of quantum gravity must clearly depend on two things. First we should know all the factors and dimensions involved in deriving $E = mc^2$. Second we must know all the factors and dimensions which were omitted and not included in the derivation of Einstein’s famous formula. When we consider the holographic boundary then it is clear that in deriving $E = mc^2$ we considered only a very minor number of the 336 dimensions representing $SL(2,7)$ and indirectly identified with the number of strong interaction particles at very high energy, i.e. gluons [1,2,5]. However, from the 20 component or degrees of freedom of pure gravity we used only 4 spacetime dimensions for the Lorentzian transformation. The ratio of the involved dimensions of those omitted or unused dimensions determines therefore $\gamma_H$ in a wider sense than that envisaged by Nottale’s scale relativity as a mere logarithmic scaling paralleling the golden mean scaling [4, 11-14]. Then it is self explanatory that the ratio we are seeking is given by

$$\gamma_H = \frac{R^{(4)} - D^{(4)}}{SL(2,7) + (R^{(4)} - D^{(4)})} .$$

Thus we have
\[ \gamma_H = \frac{1}{(336+16)/16} = \frac{1}{22}. \]

This is a clear-cut approximation of \(1/(22 + k)\) and has an obvious intuitive physical interpretation which we could phrase as follows: If we consider that the 26 bosonic dimensions of the Heterotic string theory is what describes all fundamental interactions and unifies relativity with quantum mechanics, then we did not take for \(E = mc^2\) more than 4 dimensions. The rest, namely \(26 - 4 = 22\), leads to the factor \(1/22\) of what understandably became known as missing dark energy. We know now where this missing dark energy came from. It came from the 22 dark dimensions. In addition to being compactified, meaning to be curled and thus curved even in the absence of matter, they play the role of an effective cosmological constant. A comparison between the bulk and the holographic boundary analysis of \(\gamma_H = 1/22\) is given in Table 1 where the Lorentz factor is \(\gamma_B = \gamma_H\).

Table 1: A comparison between the bulk and the holographic boundary analysis of \(\gamma_H = 1/22\) where the Lorentz factor \(\gamma_B = \gamma_H\).

<table>
<thead>
<tr>
<th>Bulk</th>
<th>Holographic boundary</th>
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<tbody>
<tr>
<td>(E_8E_8 = 496)</td>
<td>(</td>
</tr>
<tr>
<td>(\gamma_B = \frac{1}{\sqrt{</td>
<td>E_8E_8</td>
</tr>
<tr>
<td>(\gamma_B = \frac{1}{\sqrt{496-12}})</td>
<td>(\gamma_H = \frac{20 - 4}{336 + (20 - 4)})</td>
</tr>
<tr>
<td>(\gamma_B = \frac{1}{\sqrt{484}})</td>
<td>(\gamma_H = \frac{16}{336 + 16})</td>
</tr>
<tr>
<td>(\gamma_B = \frac{1}{\sqrt{(22)^2}})</td>
<td>(\gamma_H = \frac{1}{(352 / 16)})</td>
</tr>
<tr>
<td>(\gamma_B = \frac{1}{22})</td>
<td>(\gamma_H = \frac{1}{22})</td>
</tr>
</tbody>
</table>

5.The clopen universe which is its own multiverse – supersymmetric Penrose tiling

The Penrose fractal tiling is basically two things [1,20,24]. First it is a quotient noncommutative space and a two-dimensional projection of a topological 4-dimensional fractal-like space with a dimension

\[ \langle n \rangle = 4 + \phi^3 \]

where \(\phi = (\sqrt{5} - 1)/2\), i.e. a Conway-Penrose isomorphic length

\[ \ell = \langle n \rangle / 2 \rho = 2.118033989 \]

Second it is a Regge “triangulation” quantum gravity solution using 4D quasi-crystal [24]. Because of the compactified form of SL(2,C) and the corresponding Penrose universe we have an open but also closed universe. To use the topological mathematical term we should say it is clopen [36]. Again, because of the Conway-Penrose isomorphic length we have a multiverse inside a universe. That way we unified all the contradictory notions which were initially posed as exclusion propositions. Our
universe is clopened [36] and its own multiverse. Now going one step further by introducing a super symmetric Penrose universe, we can easily reason that tiling space with 11-dimensional quasi-crystals lead to an isomorphic length equal $11 + \phi^5$. Consequently the dimensions are $(2)(11 + \phi^5) = 22 + k = 22.18033989$ and the corresponding energy mass relationship is exactly the one we just obtained in the preceding section 3, namely

$$E = \frac{mc^2}{22.18033} \simeq \frac{(mc^2)}{22}.$$

Note that all the 22 extra dimensions are related to the Empty Cantor Set with negative Menger-Urhyson topological dimensions and constitute therefore a sort of negative pressure resembling Einstein’s cosmological constant.

6. Conclusion

In the last quarter of a century, Superstrings, M-theory, F-theory as well as the theory of varying speed of light [31] and scale relativity [4] besides the fractal Cantorian spacetime theories [6-14] changed many things which most of us thought of as being of unlimited validity and would never change. However, there is a method to what may appear to be the madness of change. The authors are confident that the velocity of light has forever lost its prominence and only for the better for our theoretical tower of higher and higher aims for deeper and deeper understanding. J. Magueijo, L. Smolin [31], L. Nottale [4] and G. Ord [32] were able in different but equivalent ways to admit infinite velocities and even infinite energies without destroying Lorentzian invariance and added Planckian invariance. Ord recognized from his subtle analysis of the quantum random walk that special relativity harbors quantum mechanics and that there is no absolute nor even ordinary relativistic time since time is path dependent at least in a fractal spacetime or with an anti-Bernoulli process characterized by 0, 1 and $-1$. El Naschie recognized some time ago that Hardy’s entanglement [33-35]

$$P(\text{Hardy}) = \phi^k$$

where $\phi = (\sqrt{5} - 1)/2$ is generic and fundamental [33-35]. There is little surprise then that for everything to fit seamlessly following the principle of maximal harmony that the crisp integer dimension of E8E8 is not 496 but 496 – $k^2$ where $k = 2 \phi^5$. Similarly SU(2) is not simply 3 but 16k and $\bar{c}_n$ is not 137 but $137 + k_n$ where $k = \phi^3(1 - \phi^5)$ and $k_n = \phi^5 (1 - \phi^5)$. Hardy’s entanglement $\phi^k = k/2$ is affecting everything [33-35] and it is no surprise, bearing in mind the careful Lorentzian coordinate leads to a modification of

$$E = mc^2$$

to

$$E = \gamma_H mc^2$$

where $\gamma_H$ is simply half the value of Hardy’s generic quantum entanglement [33,35] and equal to the reciprocal value of the dark dimensions left out from the 26 + k dimensions of Heterotic string theory when we subtract the 4 spacetime dimensions of relativity theory. That means

$$\frac{1}{2} P(\text{Hardy}) = 1/(22 + k) = \phi^5 / 2$$
There are many ways to show where the missing dark energy came from. However, all these different ways are different sides of the same coin. The reader is urged to ponder the beauty of self-similarity between

\[ E = \frac{mv^2}{2} \quad (\text{Newton}) \]
\[ E = mc^2 \quad (\text{Einstein}) \]
\[ E = \left(1/22.18033\right) mc^2 \quad (\text{El Naschie}) \]

and the magnificent cosmic mathematical harmony behind it [37, 40]. In conclusion we should mention an extremely important result which was obtained virtually few weeks ago by the first author namely the dark energy is the energy of the Hawking-Hartle wave of the universe with density equal to

\[ E = \left(5\phi^2 / 2\right) mc^2 \simeq mc^2 \left(21 / 22\right). \]

It follows then that cosmic measurement will collapses the Hawking-Hartle quantum wave that is unless we invent Non demolition measurement instrument. Should we be able in the future to do that then we could develop in future dark energy nuclear reactor with undreamed potential.

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References


Quantum scale voids in the fractal fabric of spacetime as origin of the hypothetical dark energy –Topological elucidation and exact analytical determination

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Abstract

The only rational explanation for the theoretically and experimentally well founded quantum entanglement is a measure zero geometry for the quantum micro spacetime, such as a higher dimensional completely disjoint version of a Menger sponge fractal quasi manifold. A spacetime of that type will necessarily contain infinitely many voids. In addition to arrive at the generic probability of quantum entanglement given theoretically and confirm experimentally to be equal to 9.01699 percent, spacetime manifold must be not only Cantorian fractal but also a random Cantor set. In other words it must obey all the postulates and rules of M. El Naschie’s E-Infinity Theory. The present work is aimed at proving El Naschie’s thesis regarding the discrepancy between the energy content of the universe predicted by Einstein’s equation (E = mc² where E is the energy, m is the rest mass and c is the velocity of light) and experimental findings that only 4.5% of this energy could be measured. This said discrepancy will be shown here following El Naschie’s pioneering work to be caused by the quantum scale voids in the fabric of spacetime itself and the intrinsic quantum entanglement at these scales which amount to the generic expectation value of Hardy, namely < P > = φ⁵ where φ = (5√5 − 1) / 2. This fusing of Einstein’s energy-mass equivalence equation with the nonlocal quantum entanglement via a suitable light cone velocity results in a substantial reduction of the theoretically predicted energy contained in the entire cosmos. From calculation one can mathematically deduce that the hidden dark energy is a staggering 95.495028 percent of total energy predicted in the conventional calculation. The agreement between our and El Naschie’s results and that of the WMAP measurement (about 4.5% for ordinary matter and 95.5% for dark matter and dark energy) is remarkable if not astounding and unprecedented.

We conclude that dark energy is a crucial and vital effect due to the nonclassical zero measure nature of the Cantorian-fractal geometry of space which manifests itself via Hardy’s quantum entanglement and leads to a modification of the conventional equations of the classical theory of relativity.

Keywords: Dark energy, Quantum fractal voids, Hardy’s quantum entanglement, Quantum gravity.

1. Introduction

By any standard the discrepancy between the theoretically predicted total energy of our cosmos using Einstein’s equations compared to the mere 4.5 percent found using highly advanced measurements is sufficient to call many of the secured pillars of theoretical physics into question [1, 2]. On the other hand there are two fundamental theories governing physics, relativity and quantum mechanics. The famous equation E = mc² of Einstein [22, 26] is based only on relativity and may frequently be wrongly interpreted outside its range of validity [22, 26]. In fact we could go even further than that.
and agree with what a leading relativity researcher, W. Rindler wrote some time ago [22]: “Nevertheless Einstein’s assertion of the full equivalence of mass and energy, according to the famous formula \( E = mc^2 \) is in part a hypothesis as we shall see. It cannot be uniquely deduced from the basic law of the conservation of our momentum”.

In fact, and as is well known, particle physicists prefer to use different versions of the formula where the concept relativistic mass is not explicitly involved [22, 26]. In the present work we will re-derive Einstein’s energy-mass relationship following Elnaschie in a way permitting the inclusion of the fundamental quantum entanglement by accounting for the special topology of the zero measure Cantorian-fractal manifold replacing the smooth 4D spacetime of special relativity [7, 10, 15, 16, 17, and 18].

Note that the present quantum entanglement relativistic mass-energy formula arrives at a vital quantum gravity equation via a measure theoretical shortcut and appropriate Lorentzian coordinate transformation of a light cone velocity usually used in the quantization of string theory [26]. However a different interpretation of the above could be more advantageous or at least complimentary for the understanding of our result and that is to invoke our earlier picture about an informational field [8, 23]. We recall that the quanta of this field were christened ‘Informions’ [8]. This was done in 1998 within the framework of Elnaschie’s concept of conjugate complex time and the new possibility for the Informions to speed up when they lose energy without acquiring complex mass-like tachyons [8]. Thus the two different interpretations namely that of a zero measure Cantorian spacetime at the root of quantum entanglement [27,28] on the one side and the informational relativistic field with particles obeying reversed energy dissipation on the other side, together give a water tight topological and physical explanation for the apparent accelerating expansion of the universe as well as the reason for measured ‘perception’ that over 95% of the energy of the universe is missing or in the form of dark energy [1, 2].

2. Analysis

Three main additional features are essential for deriving the relativistic quantum entanglement, i.e. zero measure Cantor set based energy-mass relation as compared to the derivation of the conventional expression \( E = mc^2 \). These are light cone velocity [26], conjugate complex time [8, 23] and Sigalotti’s relativistic critical velocity [14, 23].

2.1 Lorentzian coordinate transformation and light cone velocity

Unlike conventional derivation we need here the light cone relativistic quantization coordinates employed frequently in string theory. These are [26]

\[
x^+ = \frac{1}{\sqrt{2}} (x^0 + x^1)
\]

and

\[
x^- = \frac{1}{\sqrt{2}} (x^0 - x^1).
\]

Introducing the velocity parameter \( \beta \) in the usual way we find the light cone velocity [26]

\[
\frac{dx^-}{dx^+} = \frac{1 - \beta}{1 + \beta}
\]

which bypasses the usual restriction making it in principle possible to become infinite and thus more Newtonian. That way we arrive again at a familiar expression [26]

\[
E = \gamma mc^2
\]
where $\gamma$ is only formally similar to the Lorentz factor but in fact it is required to take three values, namely $\gamma = 1/2$ for $c = v$ to give Newtonian mechanics, $\gamma = 1$ to give the familiar Einstein relation and finally $\gamma = \gamma_c$ which is the Cantorian or quantum entanglement Lorentz factor which gives us a reduction of Einstein’s classical energy amounting to 95.491% in agreement with observations as mentioned above in the Introduction.

2.2 Conjugate complex time, Informions and anti-energy

Following earlier work due to Argyris, El Naschie, Sigalotti and others we now introduce Elnaschie’s conjugate complex time [8]

$$T = 0 \pm i t$$

where $i = \sqrt{-1}$ and $T = t$ and $t$ is the ordinary time. That way we find the imaginary wave length [8]

$$\lambda_{1,2} = \pm \frac{i \hbar}{\mu m_0} \sqrt{(1 + (v/c)^2)}$$

Taking the modulus of $\lambda$, one finds [8]

$$|\lambda| = \sqrt{\lambda_1 \lambda_2}$$

$$= \frac{\hbar}{\mu m_0} \sqrt{(1 + (v/c)^2)}.$$  

Taking the limit for zero mass and for infinite velocity we obtain [8]

$$|\lambda|_{m_0 \to 0} = |\lambda|_{v \to \infty} = \infty$$

This leads to [18]

$$E_{m_0 \to 0} = E_{v \to \infty} = 0.$$  

Thus by allowing for conjugate complex time we have effectively developed a nonclassical hypothetical particle akin to that suggested by Clay and Crouch but with real mass [8]. Elnaschie’s particle has an essential attribute of the current accelerating inflation of the observed universe because its velocity increases when it loses energy [8]. This is what we call Informions quanta of informational field of which the zero-measure Cantorian fractal spacetime manifold is a topological realization [7, 10, 16, 17]. The incredible fact is that nature seems to have adopted this mathematically consistent possibility because Hardy’s generic quantum entanglement was derived using orthodox quantum mechanics first [6-8] and verified experimentally [11-13][15] and second because the seemingly missing energy of the universe [1, 2] is a second accurately tested fact and harmonizes completely with Hardy’s quantum entanglement as we will see clearly in a while.

It is not strange that physics and information mix in this way. The work of Szilard, Wheeler and Zurek always pointed in this direction [3, 10]. The new ingredient here however is the link to Cantor sets and fractal spacetime geometry [7, 10, 17]. On the other hand when we do not work explicitly with fractals then we are normally obliged to use complex numbers. This was exactly the situation with Hardy’s entanglement and in the present situation in deriving the complex $\beta = v/c$ then taking its modulus.

2.3 The critical relativistic velocity

Motivated by complex conjugate time and instructive geometrical derivation of special relativity the role of the most irrational number of KAM theorem [7, 10, 11], namely $(\sqrt{5}-1)/2$ was discovered in special relativity by Sigalotti et al [14, 23] and was the subject of many subsequent important publications [14]. Taking this result into the complete derivation of the entanglement energy-mass
relation which is the main subject of the present work, it is found that the boost $\beta$ is essentially a complex number with a modulus leading to the Lorentz factor:

$$\gamma_c = 1 / \sqrt{491.96747755}$$

$$= (16)/(\text{dim SL}(2,7)_c + 16)$$

where $\text{dim SL}(2,7)_c = 336 + 16k$ is the dimension of the holographic boundary and 16 is the number of degrees of freedom in Einstein’s equation which is equal to the degree of freedom of pure gravity minus the four dimensions of spacetime.

2.4 The quantumly entangled relativistic energy mass relation of a Cantorian spacetime manifold

From 2.1 to 2.4 we can easily now write down EQR as

$$E_{QR} = \gamma_c mc^2$$

$$= [(16)/(336 + 16k + 16)] mc^2$$

where $k = \phi^3(1 - \phi^3)$ and $\phi = (\sqrt{5} - 1)/2$. Inserting in $E_{QR}$ we find that

$$E_{QR} = 0.04508497187 mc^2$$

For $mc^2 = 1$ the equation clearly predicts that 95.49150281 of the expected energy of Einstein 100% is missing [1, 2]. Noting the remarkable agreement between the measurements of WMAP which sets dark energy at 95.5% there can be no doubt that the above equation is the correct one for predicting the correct value for the magnitude of energy on intergalactic scales [1, 2].

3. Discussion and conclusion

Einstein’s formula of special relativity $E = mc^2$ is based entirely upon a smooth spacetime and classical Lorentzian coordinate transformation. It does not take into account any nonclassical topology let alone a Cantorian geometry with infinitely many holes in it [7, 11][15]. Neither does the formula take quantum mechanics on board nor the vital effect of Hardy’s generic quantum entanglement which is a consequence of the non-classical nature of the zero measure Cantorian geometry of actual spacetime fabric [11-13, 15]. We say actual because experiments showed that Hardy’s entanglement is real and this conclusion is reinforced again by the missing dark energy in the cosmos [1, 2]. The magnitude of this missing energy of 95.491% squares perfectly with our result and with Hardy’s entanglement which is theoretically and experimentally shown to be 9.01699% [11-13, 15]. The apparent accelerating expansion of the universe and the missing dark energy [1, 2] are both direct consequences of omitting the quantum topological nature of our spacetime and its simplification at low resolution to a smooth flat passive space in Newtonian mechanics and smooth curved manifold in general relativity [10, 24]. Quantum mechanics, despite all its interpretational short comings, did not fall victim to our geometrical and topological approximation of spacetime. This is so because conventional quantum mechanics does not make use of spacetime description. Only with a Cantorian fractal spacetime geometry could we bring gravity and quantum mechanics together to explain nature at the very small and the extremely large scales [2, 7, 10, 19]. It was normally assumed that gravity plays a minor practical role in particle physics and that quantum mechanics plays a minor role at the very large scales as noted by Elnaschie. The existence of dark energy is an example that this reasoning is frequently misguided. Over the years there was a flurry of papers advocating the fractal nature of
spacetime [5, 7, 10, 21]. These proposals were repeated in the not very distant past by R. Feynman and J.A. Wheeler [9]. These ideas were finally put into a definite mathematical framework by G. Ord [20], L. Nottale [19] and M. El Naschie [7, 9, 10, 11]. Even more recently T. Palmer [21] suggested a fractal state space and pointed out the intrinsic blindness of orthodox quantum mechanics to fractals. That is how things remained until the connection to Hardy’s quantitative result for quantum entanglement which was discovered by El Naschie, Ji-Huan He and L. Marek-Crnjac became known and all its ramifications were appreciated [10, 15, 17]. In conclusion we should mention that in his very recent work El Naschie established that dark energy is the energy of the quantum wave in five dimensions while ordinary energy is the energy of the quantum particle in the same Kaluza-Klein dimension. The sum of both is in turn equal to Einstein energy \( E=mc^2 \). In other words Einstein formula \( E=mc^2 \) is blind to any distinction between dark energy and ordinary energy [27,28].

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References


Mohamed Elnaschie’s revision of Albert Einstein’s E = mc²: A definite resolution of the mystery of the missing dark energy of the cosmos*

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Abstract

The Egyptian Engineering Scientist and Theoretical Physicist Mohamed Elnaschie has found a definite resolution to the missing Dark Energy of the cosmos based on a revision of the theory of Relativity. Einstein’s equation of special relativity \(E_0 = m_0c^2\), where \(m_0\) is the controversial rest mass and \(c\) is the velocity of light developed in smooth 4D spacetime was transferred by Elnaschie to a rugged Calabi-Yau and K3 fuzzy Kähler manifold. The result is an accurate, effective quantum gravity energy-mass relation which correctly predicts that 95.4915028% of the energy in the cosmos is the missing hypothetical dark energy. The agreement with WMAP and supernova measurements is astounding. Different theories are used by Elnaschie to check the calculations and all lead to the same quantitative result. Thus the theories of varying speed of light, scale relativity, E-infinity theory, M-theory, Heterotic super strings, quantum field in curved spacetime, Veneziano ‘s dual resonance model and Nash’s Euclidean embedding all reinforce, without any reservation, the above mentioned theoretical result of Elnaschie which in turn is in total agreement with the most sophisticated cosmological measurement. Incidentally these experimental measurements and analysis were awarded the 2011 Nobel Prize in Physics to Adam Riess, Brian Schmidt, and Saul Perlmutter.

Keywords: Dark matter, Homology of fuzzy Kähler, Betti numbers, Heterotic strings.
MSC Codes: 85A40, 83C40

1. Introduction

Special relativity presupposes a smooth spacetime with Lorentzian symmetry group invariance [1]. Quantum spacetime on the other hand is modeled via radically different geometrical realization [1-8]. In string theory, M-theory and super gravity one uses various types of Calabi-Yau and complex Kähler manifolds for spacetime extra dimensions [9-17]. Consequently requiring Poincaré invariance in a complex space with extra dimensions will most surely lead to a different energy-mass relation than the classical equation of special relativity. Should the principle of scale relativity hold, then one would expect to retrieve Einstein’s familiar formula in a scaled form [3-5]. Noting that for a continuous manifold the Betti number \(b_2\) which counts the three dimensional holes in a manifold is given by \(b_2 = 1\) and that the same Betti number for a K3 Kähler is \(b_2 = 22\), it is possible to show that \(E = m_0c^2\) may be elevated to a quantum relativity, i.e. a quantum gravity equation when scaled by \(\lambda_{QR} = b_2(S^3)/b_2(K^3) = 1/22\). This prior intuitive expectation noted first by Elnaschie was confirmed later on by him on two counts, namely first experimentally using the cosmic measurement Ries, Schmidt and Perlmutter [4].
and second theoretically using numerous sophisticated established theories, all leading to the same robust result, namely $\lambda = 1/22$.

In this paper we show following Elnaschie that for a fuzzy Kähler [10, 13], the scaling factor changes from $1/22$ to $1/(22 + k) = 1/(22.18033989)$. In addition to giving a derivation of $E_{\text{OR}} = \lambda (m_o c^2)$ where $m_o$ is the controversial rest mass and $c$ is the speed of light, we show that this result is in exquisite agreement with the cosmological measurement of COBE and WMAP as well as the analysis of certain supernovas which led to the award of last year’s 2011 Nobel Prize in Physics [4]. Based on K3 fuzzy Kähler, one can predict with very high precision that the percentage of hypothetical dark energy missing in the universe is 95.4915028 percent. This is a potentially unprecedented agreement between theory and measurement in cosmology, if not in all of theoretical physics [1]. We probably will know for sure when the Planck measurement project is completed. However this particular result of Elnaschie has in one giant leap unified many theories, old and new, and reconciled theory with measurement [16-19]. In Table 1 we summarize the results of various theories and methods dealing with the same energy reduction factor of almost $1/22$ [20,21].

2. Introduction and motives for revising Einstein’s energy-mass equation

As aptly noted by Elnaschie, an equation based entirely on a smooth space with Lorentzian spacetime invariants developed years before quantum mechanics was formulated by Heisenberg, Schrödinger and Dirac could not possibly be expected not to break down at some quantum or intergalactic scale [1-3]. In the present short paper we show following Elnaschie that the missing dark energy in the cosmos, discovered through various accurate cosmological measurements [4] is due to some basic fundamental inadequacies of applying Einstein’s celebrated equation $E = m_o c^2$ where $E$ is the energy, $m_o$ is the rest mass and $c$ is the speed of light outside its range of validity [5,6]. We thought for a long time and understandably so that gravity cannot have that crucial effect on elementary particle physics.

Similarly we thought that quantum mechanics also has very little effect on cosmology except maybe when it comes to incredibly shrinking objects such as black holes [1]. However when we started asking very deep questions regarding the unification of all fundamental interactions [7,8] we recognized suddenly that at the extreme small distance such as the Planck length ($10^{-33}$ cm) the feeble gravity becomes as strong as the other three fundamental forces, i.e. the weak force, the strong force and the electromagnetic force [1,3,7]. On the other hand we now just realize that quantum effect such as quantum entanglement has an impact on physics at extremely large intergalactic distances. It is so profound that the classical equation of Einstein $E = m_o c^2$ is off the correct result by almost 95.5% [4]. Seen with the eyes of a particle physicist this should not be that astonishing because the only degree of freedom of special relativity is a single messenger particle, the photon. By contrast the simplest model for high energy quantum physics, the standard model, needs 12 messenger particles equivalent to 12 degrees of freedom. Special relativity is highly confined by the Rayleigh theorem on Eigen value and is bound to overestimate energy levels.

In the present work we trace back the deficiency in $E = m_o c^2$ and prove that this is the case because of the real non-classical geometry and topology of the actual fabric of spacetime [2,7,8]. This nonclassical topology is essentially the cause of amplifying what we perceive as quantum effect which screens the energy by as much as 95.5% in full agreement with measurements. In particular we will show that the ratio of the two Betti numbers [9-12] fixes the homology of spacetime’s de Rham topology of smooth classical spacetime of relativity and the rugged K3 Kähler [11] based quantum spacetime of quantum gravity and give us a scaling $\lambda = b_2(\text{smooth})/b_2(\text{Kähler})$ which accounts for the 95.5% missing dark energy [4]. It is well known that the Betti numbers are topological invariants of a manifold [9,10] exactly as the dimensions and the Euler characteristics [9-12] with the added advantage that $b_2$ counts what we may call three dimensional holes (voids) [9] in the manifold (spacetime fabric). That way all the fine structures of our space are taken into account [9,13]. How this actually is done is what we will explain next.
3. Homology of a spacetime based on crisp K3 Kähler manifold [10]

Following super strings and related theories [12] we look first at the possibility of a quantum gravity spacetime based upon a K3 Kähler manifold [13]. We start with a non-fuzzy crisp Kähler then look at the fractal-like fuzzy case.

3.1 Classical non-fuzzy Kähler

We consider a K3 Kähler manifold with four complex dimensions used extensively in theories with hidden dimensions particularly super and Heterotic string theory [12,13]. The manifold is fixed by the Betti numbers which determine the Euler characteristic and the signature. In case of non-fuzzy (crisp) K3 the Betti numbers are [10,13]

\[ b_0 = b_4 = 1, \quad b_1 = b_3 = 0, \quad b_2^- = 19 \quad \text{and} \quad b_2^+ = 3. \] (1)

It follows then that the Euler characteristic is [10,13]

\[ \chi = b_0 + b_4 + b_2^- + b_2^+ \\
= 1 + 1 + 19 + 3 \\
= 24 \] (2)

while [10,13]

\[ b_2 = b_2^+ + b_2^- \\
= 3 + 19 \\
= 22 \] (3)

and the signature is [10,13]

\[ \chi = b_2^+ - b_2^- \\
= 3 - 19 \\
= -16. \] (4)

We stress once more that \( b_2 \) counts the 3 dimensional holes in K3 and will play a crucial role in our derivation.

3.2 Fuzzy, fractal-like K3 Kähler

Now we look at an even more exotic version of K3 [13]. With that we mean Elnaschie’s fuzzy Kähler which he used in earlier studies in a slightly modified form [13,14]. The Elnaschie Kähler we construct here is a fuzzy version of the one considered above. The Kähler in question is given by the same \( b_0, b_4, b_1 \) and \( b_3 \) as the previous crisp Kähler. Only \( b_2^- \) and \( b_2^+ \) which measure a sort of average number of 3D fractal voids are given by [13,14]

\[ b_2^- = 19 - \phi^6 \quad \text{and} \quad b_2^+ = 3 + \phi^3 \] (5)

where \( \phi = (\sqrt{5} - 1)/2 \). It follows then that [13,14]
\[ b_2 = (19 - \phi^6) + 3 + \phi^3 = 22 + k = 22.18033989. \]  

(6)

It is important to note the following: The small numbers \( \phi^6 = 0.05572809014 \) as well as \( \phi^3 = 0.236067977 \) and \( K = (1 - \phi^3) = 0.18033989 \) all have various physical, topological and geometrical interpretations. For instance \( \phi^6 \) is the exact value of the vital Immirzi parameter of loop quantum gravity without which nothing would fit in this theory [15]. In addition and as realized for the first time by Elnaschie \( \phi^6 \) may be viewed as the probability for quantum entanglement of three quantum particles while \( \phi^3 \) is the well known Hardy’s generic probability of quantum entanglement [16,17] for two particles which was also confirmed experimentally. The \( \phi^3 \) on the other hand is the generic probability of a Cantorian spacetime with a core Hausdorff dimension equal to \( (4 + \phi^3) = 4.44 \) and is directly related to the famous Unruh temperature as demonstrated by Elnaschie in some of his unpublished papers and lectures. Finally

\[ \frac{1+k}{10} = \frac{\phi^3}{2}. \]  

(7)

That means

\[ K = 5\phi^3 - 1. \]  

(8)

Which is a deep and useful relation utilized in various E-Infinity derivations.

4. Elevating Einstein’s relativistic equation to a quantum gravity energy-mass relation

We said that \( b_2 \) is an important homological invariant of a manifold [9-11] and that it basically counts the 3 dimensional voids in the manifold [9, 14]. For a two sphere \( S^2 \) or any connected manifold \( b_2 \) is equal to unity \( b_2 = 1 \). On the other hand for our classical Kähler \( b_2 = 3 + 19 = 22 \), and this number already indicates that this manifold is almost a Swiss cheese full of 3 dimensional holes [10,13]. Compared to the smooth \( S^2 \) manifold akin to the spacetime of Einstein, K3 has 22 times less spacetime and following general relativity, less energy. Now following, for instance, Nottale’s scale relativity principle, we could define a scaling \( \lambda \) to be:

\[ \lambda_{QR} = \frac{b_2(\text{Einstein space})}{b_2(\text{Kahler})} = \frac{1}{22} \]  

(9)

and use it to scale \( E = mc^2 \) to

\[ E_{QR} = \lambda_{QR} mc^2 = \left(\frac{1}{22}\right)(mc^2) = 0.0454545 (mc^2). \]  

(10)
This implies that the missing hypothetical dark energy is

\[
E(\text{dark}) = \left(1 - \frac{1}{22}\right) (100)
= 95.454545\%
\]  
(11)

This is extremely close to the cosmological measurement [4]. Even better still, if we use the fuzzy version we arrive at a mathematically exact equation

\[
E(\text{dark}) = \left(1 - \frac{1}{22 + k}\right) (100)
= 95.49150281\%
\]  
(12)

In fact when using the fuzzy Kähler we notice immediately a quantum mechanical interpretation of the result because

\[
E_{\text{QR}} = \left(\frac{1}{22 + k}\right) (m_e c^2)
\]  
(13)

means that

\[
E_{\text{QR}} = \frac{1}{2} (\phi^5) (m_e c^2).
\]  
(14)

However \(\phi^5\) is nothing else but Hardy’s generic quantum entanglement [16,17] so that our \(\lambda_{\text{QR}}\) may be viewed as the screening of a substantial part of the energy in the cosmos by quantum entanglement reducing the Newtonian action at distance by as much as \((1 - \phi^5/2)(100) = 95.4915\%\). Finally there is an even more immediate interpretation when we invoke string theory and M-theory. The largest number of dimensions in Heterotic string theory is 26 in the classical case [12] and \(26 + k\) in the transfinite fractal-fuzzy case [7, 14]. However we can make real measurement only via 4 dimensions, 3 space dimensions and one time dimension. Thus we have 22 hidden dimensions [12]

\[
D(\text{hidden}) = 26 - 4
= 22
\]  
(15)

or more accurately [12,14]

\[
D(\text{hidden}) = 26 + k - 4
= 22 + k.
\]  
(16)

Thus our scaling exponent is

\[
\lambda_{\text{OR}} = 1/D(\text{hidden})
= 1/22
\]  
(17)

or in the fuzzy case [13,14]
\[ \lambda_{\text{OR}} = \frac{1}{(22+k)}. \]  

(18)

Within this mental picture we could say that the missing dark energy is concealed and hidden inside the dark extra dimension \([7, 12, 14]\). It is a deep philosophical and ontological question to consider something which we cannot measure nor detect to be real or not.

### TABLE 1 Result of various theories applied to Dark Energy by Elnaschie and E-Infinity Group\([20,21]\)

<table>
<thead>
<tr>
<th>Theory</th>
<th>Mass-energy equation</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>General relativity plus holographic boundary</td>
<td>( E = \left( \frac{R^{(4)} - D^{(4)}}{\text{SL}(2,7) + (R^{(4)} - D^{(4)})} \right) mc^2 )</td>
<td>( R^{(n)} ) is the number Independent Components of the Riemannian Tensor in ( D = 4 ) or the degrees of freedom of pure Gravity in ( D = 8 ). Thus we have ( R^{(n)} = n^2(n^2 - 1)/12 ).</td>
</tr>
<tr>
<td>General relativity plus 6D Calabi-Yau manifold</td>
<td>( E = \frac{1}{(R^{(4)} + D^{(6)}) - D^{(6)}} mc^2 )</td>
<td>Calabi-Yau manifold has 6 real dimensions and is used as K3 Kähler in superstring theories. By contrast K3 Kähler has 4 Dimensions only but they are complex dimensions, not real.</td>
</tr>
<tr>
<td>Special relativity in a hyper 4D J. Huan He - Hilbert cube given by ( D = 4 + \frac{1}{1+\phi} ), ( 4 + \phi^3 = 4.2360679 )</td>
<td>( E = \frac{1}{2} \left( \frac{(1 - \phi)^2}{1+\phi} \right) mc^2 = (\phi^5/2) mc^2 )</td>
<td>We introduced on light cone speed ((1 - \phi)/(1+\phi)) as well as a light cone mass ( m(1+\phi) ) and utilize Hardy’s quantum entanglement.</td>
</tr>
<tr>
<td>Nottale’s scale relativity</td>
<td>( E = \left( \frac{1}{ln \alpha_{\text{GUT}}} \right) mc^2 )</td>
<td>Scaling as a gauge theory is an idea due to Herman Weyl. This idea leads to physical contradiction unless spacetime is a fractal devoid of any natural scale such as all non-Archimedean geometrical and P-Adic Theories.</td>
</tr>
<tr>
<td></td>
<td>where ( \alpha_{\text{GUT}} \approx 110 ) is the inverse coupling constant of grand unification of all non-gravitational forces. Thus ( E = \frac{mc^2}{(4.70042)^2} )</td>
<td>( = \frac{mc^2}{22.09} \approx \frac{1}{22} mc^2 )</td>
</tr>
</tbody>
</table>
5. Conclusion

The homology of K3 Kähler and what Elnaschie calls extra “dark” dimensions is the definite cause behind what we call the missing dark energy [4]. To arrive at the correct quantitative result and reconcile theory with experiments we need to scale the classical $E = mc^2$ by a scale relativity factor $\lambda_{QR}$ defined as the ratio of two second Betti numbers [10,11]. Since the Betti number of fuzzy Kähler $b_2$ is $22 + k$ and since $b_2 = 1$ for Einstein space of special relativity, our $\lambda_{QR}$ becomes equal to $1/(22 + k)$ and one finds $E_{QR} = \lambda_{QR} mc^2$ [9]. This means the so-called missing dark energy of the cosmos is exactly equal to $(1 - \lambda_{QR})(100) = 95.4915028\%$. It is almost surreal how close the results of cosmic measurement are to this percentage [4]. Noting that $\lambda_{QR}$ may also be written as $\phi^{5/2}$ that means half Hardy’s quantum entanglement probability found using orthodox quantum mechanics and confirmed through sophisticated quantum information experiments, we feel that the ordinary sharp non-fuzzy K3 Kähler manifold approximates quantum gravity spacetime geometry and topology to an astonishing extend and must be real. Seen that way, we must infer that the designer of the universe is a mathematician [1] with a deep inclination towards topology, geometry and number theory [7]. From a purely intuitive viewpoint however the result is not surprising when we remember that in terms of particle physics Einstein’s special relativity could in principle be found from a Lagrangian with a single generalized coordinate, namely the photon. A realistic theory of nature however must have a Lagrangian with a minimum of 12 generalized coordinates representing 12 massless gauge Bosons being the number of messenger particles of the standard model of high energy physics [1, 7, 14]. Our E-Infinity team has checked Elnaschie’s result using at least 10 different theories including Nottale’s scale relativity, Magueijo and Smolin’s varying speed of light theory, Witten’s M-theory, Veneziano’s dual resonance theory and quantum field Yang-Mills theory in curved spacetime and obtained exactly the same result reported[18-21]. Table 1 gives an overview of some of these results. With that we feel quite confident that the mystery of the dark energy has been solved at least in principle by Mohamed Elnaschie and that it is essentially not a mystery any more. Even more convincing is Elnaschie most recent result that dark energy is the energy of the quantum wave which collapses at measurement and thus preclude detection.

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Using varying speed of light theory to elucidate and calculate the exact experimental percentage of the dark energy in the cosmos

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Abstract

We use Magueijo-Smolin varying speed of light (VSL) theoretical modification of Einstein’s energy mass relation to derive an exact value for the missing dark energy which is found to be in astonishing agreement with the latest result of the WMAP measurement and the independent super nova analysis. Thus while Einstein’s formula predicts 95.5% more energy than found in highly precise astrophysical measurement, our VSL based calculation indicates an exact theoretical value of only 4.508497% real energy. Consequently the exact conjectured missing dark energy must be 95.491502%. By any standards this is an astounding confirmation for both the cosmological measurement and the VSL theory.

Keywords: Dark energy, Quantum gravity, Mageuijo varying speed of light theory, Doubly relativistic, Planck invariance.

1. Introduction

The issue of missing dark matter and worse still missing dark energy in the cosmos has now become a real colossal problem threatening the very foundations of theoretical physics and cosmology [1,2]. There is new undeniable confirmation via sophisticated analysis of WMAP and super nova measurement that approximately 95.5% of the energy which the cosmos should contain is not there to see or measure [1].

In the present work we give an exact analysis showing that the energy value predicted by Einstein’s special theory of relativity $E = mc^2$ where $E$ is the energy, $m$ is the rest mass and $c$ is the velocity of light gives the wrong answer when it comes to calculating the intergalactic energy scales [1,2]. The correct equation in this case must take quantum mechanics and nonclassical topology of the fabric of spacetime into account [2-8] in order to bring theoretical predictions and experimental measurement together. To this end we will use the equation developed by Magueijo and Smolin [9, 10] to modify Einstein’s relativity theory using the theory of varying speeds of light (VSL). Subsequently, we give the VSL equation a novel topological interpretation using an eleven dimensional theory akin to that of super gravity and M-theory but within a fractal setting [7, 8]. At the end the so-obtained result confirms the absence of 95.4915% of the energy of the cosmos as well as the experimental findings based on the WMAP and super nova measurement which indicates 95.5% dark energy [1, 2]. The amazing agreement between theory and experiment is unparalleled and leaves very little room for doubting the cosmological measurement [1] or the VSL theory [9, 10] nor indeed for that matter, the fractal nature of spacetime at the quantum scales [3-8].

2. The energy-mass relation of the VSL theory

The completely new idea behind the derivation of the energy-mass relationship using the VSL theory basically comes partially from Nottale’s theory of scale relativity [3]. Both scale relativity and VSL theory [9, 10] take the Planck energy $E_p = 1/\ell_p$ where $\ell_p$ is the Planck length as invariant.
Consequently J. Magueijo and L. Smolin arrive at their replacement of Einstein’s single equation $E = mc^2$ by three remarkable and thoroughly ingenious equations [9]

$$E_o = \frac{mc^2}{\gamma m_c^2}$$

with the usual $m$ transformation $m = \gamma m_c$ where $\gamma$ is the Lorentz factor but with $E$ and momentum $p$ transforming as [9]

$$E = \frac{m}{1 + \gamma}$$

and $$P = \frac{mv}{1 + \gamma}$$

The preceding equations give us the possibility of admitting a ratio $\frac{m_c c^2}{E_p}$ far larger than unity. In other words through the postulates of VSL theory $\frac{m_c c^2}{E_p}$ can become many times larger than $E_p$ without violating Lorentzian invariance, nor the $1/L_p$ invariance of the theory [9]. To show that this superficially contradictory statement leads to a true formula for quantum relativity and gives results in full agreement with the cosmological measure is our next and main task.

3. Dark energy for the extra dark dimensions

From the first equation of Magueijo-Smolin [9, 10] it is clear that our crucial ratio $\frac{m_c c^2}{E_p}$ could take three vital values:

(a) $\frac{m_c c^2}{E_p} \approx 0$
   i.e. we have Einstein’s classical equation of special relativity.

(b) $\frac{m_c c^2}{E_p} = 1$
   i.e. we have the Newtonian expression for energy when setting $c = v$ and find $E = (1/2) mv^2$.

(c) $\frac{m_c c^2}{E_p} = 21$.
   This is a crucial situation for us as we will explain shortly. The energy in this case could be written as $E_o = \frac{m}{(1 + 21)c^2} = \frac{m c^2}{22}$

Let us consider the above from the viewpoint of spacetime dimensions [7, 8] and in particular from the view point of the theories involving extra spacetime and compactified dimensions [7-18].

In case (a) above we could say that space and time fuse into $3 + 1 = 4$ dimensions. There are zero hidden dimensions in this theory. The total energy $E_o$ is not reduced or diluted by extra dimensions.

In case (b) above we could say, somewhat surprisingly, that we have a hidden dimension, namely time appearing only as a parameter. The energy $mc^2 \to mv^2$ is divided therefore by 2.

On the other hand in the most important case concerning the present work i.e. case (c), we could interpret the factor 21 as the number of dark dimensions of 26 bosonic string theory [7, 8, 17] when a 5 dimensional Kaluza-Klein theory is thought. Alternatively we take $1 + 21 = 22$ factor to mean the 22 compactified [17] invisible, dark and hidden dimensions of non-super symmetric string theory [16, 17]. In fact the 22 dark dimensions could be interpreted differently as 11 bosons and 11 fermions of a super symmetric 11 dimensional M-theory [11-13]. That way we have a new modified energy-mass relation almost identical to Einstein’s equation with the exception that the energy is now reduced by a factor of 1/22. This reduction amounts to diluting or spreading thinly Einstein’s energy because of the extra dimensions which exist but were not taken into account in the derivation of the 4D theory of Einstein.

Therefore we can write [9]
E_{\text{max}} = \left(\frac{m_c^2}{22}\right) \cong (0.04545) m_c^2.

This result is tantamount to saying that \((1 - 0.04545)(100) = 95.454\%\) of the energy in the universe is dark energy. The result is very close indeed to the experimental measurement of 95.5\% [1] and means that the extra dark extra dimension of micro spacetime is the cause for dispersing energy to the extent that we notice only a fraction equal to 4.5\% of the classically predicted total. On the other hand we know that fractal spacetime implies extra dark dimensions as well as nonclassical microscopic mechanics which we perceive as quantum mechanics [4-6]. Thus we will look once more at the preceding result from the viewpoint of quantum mechanics and thus quantum relativity, i.e. quantum gravity [3-5, 13, 15-17].

4. Hardy’s quantum entanglement behind dark energy

The classical Heterotic string theory [17] stipulates three dimensions, namely 26, 16 and \(26 - 16 = 10\) superstring dimensions of which 6 are compactified to give a 4 dimensional classical or relativistic spacetime [18]. Setting this theory in fractal dressing one finds 26 + k, 16 + k and \(26 + k - (16 + k) = 10k\) where \(k = 0.18033989\). Thus our hidden dark dimensions of the previous analysis are made “fractally” accurate by writing

\[26 + k - 4 = 22 + k\]

where \(k = \phi^3(1 - \phi^3) = 0.18033989\) and \(\phi = (\sqrt{5} - 1)/2 = 0.618033989\). The exact energy \(E_{\text{max}}\) is consequently

\[E = \left(\frac{1}{22 + k}\right) (m_c^2) = \frac{1}{2} (\phi^3) (m_c^2)\]

The remarkable thing is that \(\phi^3\) is the exact theoretical value which Hardy found for a generic quantum entanglement of two quantum particles [19]. This value was also experimentally confirmed with high precision and sophisticated quantum informational methods using modern beam splitters etc. [5, 6, 19-22].

In addition, the same previous result can be found directly from a delicate light cone quantized Lorentzian transformation [18] and confirms the intuitive initial hunch that \(E = (1/2) (\phi^3) m_c^2\) is a quantum gravity formula for quantum relativity. Consequently we could rephrase our hidden dark dimensions interpretation [7, 14, 15] to mean that the inclusion of quantum entanglement \(\phi^3\) of a single particle i.e. \(\phi^3/2\) is clearly the cause for the 95.5\% reduction in the classical energy prediction and therefore gives a quantum gravity explanation for the missing dark energy [12], [21,22].

5. Conclusion

We give a hidden dark dimension interpretation to the Magueijo-Smolin energy mass equation of VSL theory [9, 10]. The quantitative result fully explains the missing dark energy of the cosmos [1, 2]. Einstein’s equation predicts \(E = m_c^2\). The VSL equation on the other hand predicts approximately 1/22 of this value. Accurate calculation shows that 22 = 26 - 4 hidden dark dimensions of bosonic string theory should be 22.18033989 when taking the “fine structures” of the strings into account and consequently the exact reduction is not 1/22 but \(1/(22 + k) = \phi^3/2\) where \(\phi = 2/(1 + \sqrt{5})\). On the other hand Hardy’s quantum generic entanglement for two particles is given exactly by \(\phi^3\), a quantity which was found experimentally [5, 6, 19]. Consequently the reduction factor in the energy which is the cause for suspecting a 95.5\% missing dark energy in the cosmos is clearly due to the inclusion of the
quantum entanglement effect of a single particle in Einstein’s formula to elevate it from a relativistic formula to a quantum relativity formula predicting accurately the real energy content of the cosmos. Either way the rationale behind the dark dimensions and the quantum entanglement interpretation is the same. The infinitely many gaps in the fabric of spacetime itself are what simulate quantum entanglement or hidden dark dimensions reduction of the total energy predicted by the classical formula by as much as 95.5% [1, 2]. The present result confirms that the cosmological measurements are accurate and fully deserves the 2011 Nobel Prize and that the VSL theory of Mageuijo and Smolin is a valid and accurate theory [9, 10]. In short the final conclusion is that dark energy is the energy of the quantum wave while ordinary energy is the energy of the quantum particle [21,22].

References

A minimalistic theoretical derivation of Elnaschie’s missing hypothetical dark energy*

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Abstract

We give a mathematical derivation of Mohamed Elnaschie’s modified Einstein’s equation for the hypothetical dark energy in the cosmos which agrees with WMAP and supernova measurements.

Keywords: Dark energy of the quantum wave, Ordinary energy of the quantum particle M-Theory, Extra dark dimensions, Revising Einstein relativity, Hawking-Hartle quantum wave.

1. Introduction

As emphasized by L. Nottale, G. Ord and M. Elnaschie, the manifestly hierarchal structure of the universe [1] lends itself to the idea of self similarity [1-5] and self affinity in the sense of B. Mandelbrot [3,5] as well as scaling in the sense of G. Barenblatt [5]. In the case of the most fundamental and abstract concept in physics, namely energy, scaling takes its most direct and simple form $E \to E' = \lambda E$ where $E$ is energy and $\lambda$ is the scaling “exponent” [5]. As stressed by Elnaschie, the idea is essentially that of the original gauge theory of Hermann Weyl [6] and is related to scale relativity [7] and Planck energy invariance [8]. Based on these simple ideas as well as the structure of the standard model, Elnaschie generalized Einstein’s $E = mc^2$ where $m$ is the mass and $c$ is the speed of light to an equation which takes care of all fundamental interactions including gravity [9]. That way the mystery of the missing 95.5% hypothetical dark energy in the cosmos [10] was resolved by Elnaschie [10-16] with an almost magical simplicity. Nevertheless the work of Elnaschie draws its validity in the first instance from total agreement with the actual measurements. The analysis is based entirely upon generally accepted theories and leads to a solution for a major problem for which there is at present as good as no solution, let alone a theory which could be called mainstream. The contribution of the Egyptian Mohamed Elnaschie is in any event the first theory for dark matter except for the reinterpretation of Einstein’s cosmological constant [1, 16].

2. Mathematical derivation

The main arguments of our present mathematical derivation which follow the spirit of Elnaschie’s pioneering idea consist of about seven main steps which we present sequentially as follows:

(1) In terms of particle physics $E = mc^2$ is based on a single messenger particle, the quanta of the electromagnetic field, namely a boson called photon [11, 12]. At the inception of special relativity,

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* This paper is dedicated to Prof. G. Ord, Prof. L. Nottale and Prof. M. Elnaschie, the three pioneers of the theory of fractal spacetime

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quantum mechanics and the standard model of high energy particles were not invented or discovered yet. Thus Einstein committed no mistake, because neither he nor anybody else knew better at that time.

(2) The standard model on the other hand includes 12 messenger particles. Thus while the photon is represented by a Lie symmetry group U(1) with \( \dim U(1) = 1 \), the standard model is represented by the combined Lie symmetry groups \( SU(3) \) \( SU(2) \) \( U(1) \) \([11,12]\). The dimension of this group is obviously \( 8 + 3 + 1 = 12 \). These are exactly 12 isometries creating 12 particles, as per E. Noether’s theorem\([17]\). In addition they could be viewed as generalized coordinates, i.e. degrees of freedom in a corresponding effective quantum gravity Lagrangian.

(3) As is well known gravity is not included in the standard model. However it can come in from a backdoor via super symmetry of the standard model \([11, 12]\) like super gravity and M-Theory \([17]\).

(4) Since we included in special relativity only 1 of 12 photon-like messenger particles, it follows that the effect of \( 12 - 1 = 11 \) particles plus gravity was not taken into consideration at the beginning of the last century when Einstein drove his \( E = mc^2 \) \([1-9]\).

(5) Thus if we want to transform \( E = mc^2 \) from an equation of special relativity \([13,14]\) to an equation of effective quantum gravity \([15]\) then we must scale \( E \) using the simplest possible scaling exponent. In fact it is more of a multiplicative factor than exponent in this case, similar to Lorentzian boost.

(6) Elnaschie correctly argued that the simplest scaling “exponent” is obviously the ratio of the special relativity “weak” theory to the effectively quantum gravity “strong” theory

\[
\lambda = \frac{1\text{ photon}(\gamma)}{11\text{ particles} + 1\text{ sparticle}} = \frac{1}{22}
\]

where the 11 sparticles refer to super symmetric partners \([11]\) which indirectly bring in gravity as said earlier on in Step 3 \([17]\). This 22 could also be pictured as what remains from the 26 Bosonic string dimensions when we subtract the 4 spacetime dimensions and are left with 22 “dark” dimensions \([18]\).

(7) Elnaschie’s equation of “quantum relativity” for energy follows from the above as

\[
E_{QR} = \left( \frac{1}{22} \right) mc^2 = 0.04545 mc^2.
\]

This is only 4.5% from what it was supposed to be according to special relativity and matches the observations made by WMAP and supernova analysis with incredible accuracy \([10, 16]\). That way the missing hypothetical dark energy was explained in principle as well as quantitatively \([16]\).

(8) The most important realization now is that the 22 dark dimensions play the same role as Einstein’s cosmological constant and produce a negative pressure increasing expansion of the Universe as pointed out by Elnaschie in his memorable lecture in Alexandria in 2012 \([19-21]\).
(9) In his most recent work Elnaschie demonstrated the dark energy is the energy of the quantum wave since measurement collapses the wave [20,21]. Only Nondemolition measurement instrument could detect dark energy if at all possible.

3. Conclusion

In conclusion we may mention that the same result obtained here was reproduced by Prof. Ji Huan-He and Prof. Mohamed Elnaschie using sophisticated modern topological methods which space limitation precludes from giving here in detail. For instance $\lambda$ could also be defined as the ratio of the second Betti number of the spacetime of special relativity $b_2 = 1$ to the Betti number of a K3 Kähler complex manifold $b_2 = 22$ so that $\lambda = 1/22$ again. In fact it is possible to use an effective simple Lagrangian with 12 generalized coordinates corresponding to the 12 elementary particles of the standard model to reason that the special theory of relativity will grossly over estimate energy. This follows from Rayleigh’s theorem on Eigen values as explained by Elnaschie. In fact for the degrees of freedom $n \to \infty$ leads to $E \to 0$. However what we found here is an expectation value of what we call quantum relativity energy which strictly speaking should be written as $<E_{QR}> = 0.0454 \, mc^2$. Furthermore we could introduce a gauge fixing based on the normalized sum of all inverse coupling of all fundamental interactions which is equal to 100 as given by E-Infinity theory. That way we can reason that the integer value of the Baryon is 4% having the same weight as the dimensions of Einstein’s spacetime. Dark matter would then correspond to 22 dark dimensions, equal 26 Bosonic dimensions minus 4. This is then the 22% dark matter and we are left with $100 - (22 + 4) = 74$ percent hypothetical dark energy. This last conclusion depends crucially upon the correspondence between Noether’s conservation laws, isometries, i.e. internal and external symmetries and particle-like quantum states in addition to gauge invariance, all of which are fulfilled within Elnaschie’s analysis [7, 8, 18]. In all events a special merit of our analysis is that it contains no element of speculative thinking apart from fitting perfectly to actual cosmological measurements [1, 16]. Elnaschie most important conclusion however is that dark energy is the energy of the quantum Hawking-Hartle wave of the universe and since measurement collapses the quantum wave, it cannot be measured unless we develop in future some Nondemolition measuring instruments.

References


Missing dark energy: A variation on Mohamed Elnaschie’s irreducibly simple argument for its definite theoretical derivation

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The present short note is intended as a tribute to Mohamed Elnaschie’s achievement of a revision of Einstein’s famous equation and subsequently a resolution of dark energy. The manifestly hierarchical structure of the universe [1] lends itself to the idea of self similarity [1-5] and self affinity in the sense of B. Mandelbrot [3, 5] as well as scaling in the sense of G. Barenblatt [5]. Thus nonlinear dynamics, fractals and self similarity were the starting point of Mohamed Elnaschie and the key to his successful resolution of Dark Energy [19]. In the case of the most fundamental and abstract concept in physics, namely energy, scaling takes its most direct and simple form $E \rightarrow E' = \lambda E$, where $E$ is energy and $\lambda$ is the scaling “exponent” [5]. The idea is essentially that of the original gauge theory of Hermann Weyl [6] and is related to scale relativity [7] and Planck energy invariance [8]. Based on these simple premises as well as the structure of the standard model, one could generalize Einstein’s $E = mc^2$ where $E$ is energy, $m$ is the mass and $c$ is the speed of light to an equation which takes care of all fundamental interactions including gravity [9]. That way the mystery of the missing 95.5 hypothetical dark energy in the cosmos [10] was resolved by Elnaschie [10-16] with an almost surreal simplicity. I go step by step in the natural logical order and classical tradition of the famous Egyptian and Greek scientific thinking which flourished in Alexandria:

Step 1:
We recall that in terms of particle physics $E = mc^2$ is based on a single messenger particle, the quanta of the electromagnetic field, namely the photon [11,12]. At the inception of special relativity quantum mechanics and the standard model of high energy particles was not invented or discovered yet.

Step 2:
The standard model by contrast includes 12 messenger particles. Thus while the photon is represented by a Lie symmetry group $U(1)$ with dim $U(1) = 1$, the standard model is represented by the combined Lie symmetry groups $SU(3)SU(2)U(1)$ [11,12]. The dimension of this group is obviously $8 + 3 + 1 = 12$. These are exactly 12 isometries creating 12 particles, as per E. Noether’s theorem [17].

Step 3:
Gravity can come in the standard model from a backdoor via super symmetry of the standard model [11,12] like super gravity [17].

Step 4:
Since Einstein included in special relativity only 1 of 12 photon-like messenger particles, it follows that the effect of $12 - 1 = 11$ particles plus gravity was not taken into consideration at the beginning of the last century when Einstein drove his $E = mc^2$.

Step 5:
Thus if we want to transform $E = mc^2$ from an equation of special relativity [13,14] to an equation of effective quantum gravity [15] then we must scale $E$ using the simplest possible scaling exponent.
In fact it is more of a multiplicative factor in this case, similar to Lorentzian boost. In this manner, Elnaschie is running energy as we run coupling constants in quantum field theory.

**Step 6:**
The simplest scaling “exponent” is obviously the ratio of “weak” theory to the “strong” theory

\[
\lambda = \frac{1\text{photon}(\gamma)}{(12-1)\text{particles} + (12-1)\text{sparticles}} = \frac{1}{22}
\]

where the 11 particles refer to super symmetric partners [11] which indirectly bring in gravity as said earlier on in Step 3 [17]. This 22 could also be pictured as what remains from the 26 Bosonic string dimensions when we subtract the 4 spacetime dimensions and are left with 22 “dark” dimensions [18].

**Step 7:**
As noted by many authors including Elborei, the head of Physics Department in the University of Alexandria, Elnaschie’s equation of “quantum relativity” for energy follows from the above as

\[
E_{QR} = \left(\frac{1}{22}\right) mc^2 = 0.04545 mc^2.
\]

This is only 4.5% from what it was supposed to be according to special relativity and matches the observations made by WMAP and supernova analysis with incredible accuracy [10,16]. That way the missing hypothetical dark energy could be explained [16]. In conclusion we may mention that the same result obtained here was reproduced by Prof. Elnaschie using sophisticated modern topological methods. For instance \(\lambda\) could also be defined as the ratio of the second Betti number of the spacetime of special relativity \(b_2 = 1\) to the Betti number of a K3 Kähler complex manifold \(b_2 = 22\) so that \(\lambda = 1/22\) again.

In conclusion I like to thank Prof. Ji Huan-He for organizing this great and historical day in Shanghai. Last but not least I thank again all the members of the E-Infinity Group for their stimulating scientific discussions and Prof. Elborei of Alexandria University for providing material from his excellent conference on Elnaschie’s work [19-21].

**References**


Elnaschie’s resolution of the mystery of missing dark energy of the cosmos via quantum field theory in curved spacetime

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Abstract

The basic idea of quantum field theory in curved spacetime is employed following a proposal by the Egyptian scientist Mohamed Elnaschie to derive a new so-called energy mass relation of the type \( E = \gamma mc^2 \) where \( \gamma \) is a reduced Lorentz factor, \( m \) is the mass and \( c \) is the velocity of light. Although quantum field in curved spacetime is not a complete quantum gravity theory, Elnaschie’s prediction here of 95.4545% dark energy missing in the cosmos is in near complete agreement with the WMAP and supernova measurements as well as the exact solution using the methodology of Elnaschie’s E-Infinity theory.

Keywords: dark energy, mass-energy relationship, E-infinity theory, Yang-Mills theory.

1. Introduction

While special relativity predicts \( E_{\text{max}} = mc^2 \) is the 100% energy contained in the cosmos, various sophisticated measurements conducted over a long period of time such as COBE, WMAP and supernova measurements and analysis clearly indicate that nearly 95.5% of the 100% of this energy is not there [1]. The issue of the missing dark energy therefore has serious consequences for the very foundations of both cosmology and theoretical physics [1-12].

In the present work we follow a proposal by Prof. M. Elnaschie and use the basic concepts of a rather successful although only approximate theory, namely quantum field theory in curved spacetime [2, 3] to revise Albert Einstein’s energy equation [6-10]. The energy estimated using the new equation is surprisingly very close to the results of actual cosmic measurement [1] and confirms the conclusion of Elnaschie that Einstein’s equation \( E = mc^2 \) is being used outside of its range of validity and the assumption underlying its derivation when applied to the entire cosmos [6-10]. We start by giving a very concise discussion of quantum filed theory in curved spacetime [2, 3] and Yang-Mill’s theory [4-6]. Subsequently we derive the new mass energy relation.

2. Quantum field theory in curved spacetime (QFCS)

It is definitely an educated guess to presume that an accurate estimate of the magnitude of the entire energy in cosmos needs a theory for quantum gravity [6]. Quantum field theory in curved spacetime on the other hand is very useful in situations where the quantum nature of a field and gravitation are both important but not in the case when the quantum nature of gravity is crucial [6]. Thus quantum field theory in curved spacetime is just a reasonable approximation to a fully fledged quantum gravity.
theory [6]. Therefore in QFCS one treats gravity itself classically within the framework of general relativity. Thus we will have the same number of independent components of the Riemann tensor in 4D space, namely [7]

\[ R(n = 4) = n^2 (n^2 - 1)/12 = 20 \]  

as well as the number of Killing vectors field, namely [7]

\[ N_k(n = 4) = n(n + 1)/2 = 10 \]

Nevertheless and as we will see in what follows, the result of Elnaschie obtained using QFCS [1,3] for the missing dark energy [1] is surprisingly very accurate when compared to other more exact theories ad more importantly, when compared to the actual accurate cosmological measurements [1, 9, 10]. Now we turn our attention to the backbone of quantum field theory, namely Yang-Mills theory [4,5].

3. Yang-Mills theory and its three photons

Gauge invariance is fundamental to quantum field theory and Yang and Mills proposed in their famous theory to extend the set of possible gauge transformation [4, 5]. Thus a potential field with 12 rather than 4 components was introduced. Consequently one finds that in Yang-Mills theory we actually have not one photon but three, two of which are electrically charged [4, 5]. The neutral photon is our ordinary photon and should not be confused with Z^{-} of the electroweak [7]. The critical point is that these are really massless photons. Massive photon-like rho-resonance can be charged without problems but there the situation is different. In any event and as noted by Elnaschie [9, 10] we now have two more photons than in Einstein’s theory of relativity. In the next section we will see how Yang-Mills theory and Einstein’s classical relativity could be combined to use as a quantum Yang-Mill theory in curved spacetime [2-5] to produce an energy mass formula capable of dealing with the issue of the missing dark energy of the universe [1, 9, 12].

4. Yang-Mills plus Einstein’s gravity

Elnaschie subsequently reasoned that it is now a trivial observation to realize that while Einstein’s special relativity formula depends upon a single elementary particle, the photon, a fusing of general relativity with Yang-Mills quantum electrodynamics will depend upon 23 degrees of freedom [9, 10]. These are the 20 independent components of the Riemann tensor in 4 dimensions or equivalently the 20 degrees of freedom of pure gravity in 8 dimensions [7]

\[ d^{(n=8)} = (n)(n - 3)/2 = (8)(5)/2 = 20 \]

and the additional 3 photons [4,5] so that the total is 20 + 3 = 23.

On the other hand only the familiar neutral photon was used to produce \( E = mc^2 \). Consequently the additional degree of freedoms which did not enter into the derivation of \( E = mc^2 \) are \( 23 - 1 = 22 \). This is not surprisingly equal to the number of what Elnaschie calls dark dimensions of string theory when we subtract our familiar \( D^{(4)} = 4 \) from the 26 dimensions of Bosonic string theory [7]. Invoking Weyl scaling and Nottale’s scale relativity principles [8] it is again an educated guess
which can be substantiated via a watertight mathematical derivation that \( E_{\text{max}} = \gamma mc^2 = mc^2 \) where \( \gamma = 1 \) should now be replaced by \( \lambda_s = 1/22 \) as follows

\[
E_{\text{max}} = \gamma mc^2 \\
= \frac{1}{22} mc^2 \\
= \gamma mc^2
\]

where the Lorentz factor also becomes \( \gamma = 1/22 = 0.0454545 \). This is a reduction in \( E_{\text{max}} \) by 95.4545% in astonishingly accurate agreement with the result of cosmological measurements \[1\]. Following El naschie the 22 in the Lorentz factor could be deduced and interpreted in a variety of ways \[9, 10\]:

a. It is the ratio of the second Betti number (\( b_2 \)) of the connected four dimensional spacetime of special relativity (\( b_2 = 1 \)) to that of a K3 Kähler used for superstring and M-theory compactification (\( b_2 = 22 \)), i.e. \( = 1/22 \).

b. It is the number of “dark” dimensions left from the 26 dimension of Bosonic strings after subtracting our familiar \( 3 + 1 = 4 \) dimensions of classical relativity (\( 26 - 4 = 22 \)).

c. It is the inverse of the square root of the dimensionality of \( [E8E8] = 496 \) exceptional Lie symmetry group of Heterotic string theory after subtracting the 12 massless gauge bosons of the standard model, i.e. \( \gamma = 1/\sqrt{496 - 12} = 1/22 \).

### TABLE 1

| VSL Theory of J. Magueijo and Lee Smolin | The Original Formula is \( E = \frac{mc^2}{1 + mc^2 / E_p} \) 
It is transformed to \( E = \frac{mc^2}{1 + \sqrt{1 + [SL(2,7)] + R^6}} = \frac{1}{22} mc^2 \) |
| SL(2,7)= 336 is the classical Holographic boundary i.e a Klein modular curve. \( R^{(6)} \) is the number of independent components of Riemannian Tensor in \( D = 6 \) where \( D^{(10)} - D^{(4)} = D^{(6)} = 6 \). This means: \( R^{(6)} = 6^2 (6^2 - 1) / 12 = 105 \) |
| The same result is found as the Modular space dimension \( M(5, 21) = 105 \) and also an approximation to \( \alpha_{\text{GUT}} = 105 \) of Grand Unification. |
| E12 conjectured exceptional Lie Group and \( cp^{(4)} \) Calabi-Yau Manifold with Euler characteristic equal 200. | \( E = \frac{mc^2}{\sqrt{|E_{12}| - \chi}} \) = \( \frac{mc^2}{\sqrt{(57)(12) - 200}} = \frac{mc^2}{\sqrt{684 - 200}} = \frac{mc^2}{22} \) |
| \( |E12| = 684 \) is very Close to \( \sum_{\text{Stein}} = 686 \) where Stein stands for the dimensions of two and three stein spaces. There are only 17 of them. |
5. Conclusion

Following El Naschie in combining classical general relativity with Yang-Mills gauge theory we produce an effective gravity-quantum field theory which is a first order approximation to a possible quantum gravity theory [6]. Although the theory is only an approximation, the results for the missing dark energy obtained using this theory are more than excellent compared to those obtained using far more complex analyses such as superstring theory and the homology of Kähler manifolds. The most important point however is that El Naschie’s theoretical result explained here agrees completely with cosmic measurement from COBE to WMAP [1]. In addition, our E-Infinity research group has used about 15 different theories to ascertain the correctness of El Naschie’s insight and all have given essentially the same result presented here [9, 10]. A sample of these various results is given in Table 1. The reader should note the ingenuity of El Naschie as well as Magueiho and Lee Smolin’s theories in that they preserved the vital Lorentz invariance while allowing for varying speed of Light [9-12]. Finally we should mention El Naschie most recent result that dark energy of the cosmos is the energy of the Hawking-Hartle quantum wave which collapses at measurement with the nature outcome that we cannot detect or measure dark energy.

Acknowledgement: We are grateful to Prof. El Naschie for lengthy discussions over many years regarding the content of this paper. The work is supported by PAPD (A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions), National Natural Science Foundation of China under Grant No.10972053

References

Spacetime and the missing uncertainty*

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Abstract

The classical concept of spacetime is strongly motivated physically, arising as it does from Einstein's two postulates and some assumptions about worldlines and events. The concept itself gives no hint that quantum mechanics is somehow hidden from view. In this talk we compare features of spacetime and quantum propagation, showing that both have a common progenitor in discrete relativistic processes, but differ in their treatment of the transition from digital to analog time.

1. Introduction

Minkowski spacetime as a concept seems so strongly rooted in Einstein's two postulates that it appears to be more than just a mathematical convenience. Since Minkowski's time, it has become common to think of spacetime as a physical entity, a container similar to Newton's space-time, in which physical objects persist. Compared to the relatively abstract and non-physical assumptions required by the foundations of quantum mechanics, the spacetime picture would seem to be more physically cogent and robust than the quantum models that followed it [1].

The physical appeal of Minkowski spacetime suggests we may use it to probe questions about quantum mechanics. After all, if we believe that Minkowski space is firmly established as a model of physical 'reality', we can use it to see how its formulation might overlook the quantum mechanics we know must be hidden.

Comparing relativity and quantum mechanics in general, relativity is about events assumed to be points in a 4-dimensional spacetime. Point particles are assumed to have smooth worldlines and "Spacetime tells matter how to move." [2] Spacetime 'contains' particles and 'informs' them. Although mass is a background feature in special relativity, it acts back on spacetime in general relativity "Matter tells spacetime how to curve." [2] suggesting that special relativity omits a dynamical component of matter.

In quantum mechanics, events are not spacetime points. The closest quantum mechanics comes to the objective event of relativity is the 'measurement'. Furthermore, while mass appears to be a background feature in non-relativistic quantum mechanics, 'zitterbewegung' in the Dirac equation suggests that mass may have a dynamic origin.

The differing concepts of events in relativity and quantum mechanics become even more marked when one considers worldlines vs. paths. Worldlines are assumed smooth for free particles whereas quantum paths are known to be Fractal. It is this latter aspect of paths that contributes to the uncertainty principle [3-5], a recognized harbinger of quantum mechanics.

The smooth worldlines of special relativity are incompatible with the uncertainty principle so it is this aspect of relativity that we shall explore. We shall investigate how simple digital relativistic clocks keep track of time, and consider different possibilities for continuum limits. We shall follow

* This is an adaptation of a talk given at the 2012 International Symposium on Non-linear Dynamics in Shanghai, China, via Skype.
models of clocks developed in [6-9].

2. Digital to Analog Transitions in Clocks

Consider a model clock that ticks once every unit of time (Fig. 1A). In a two-dimensional spacetime, the forward and backwards light cones of each tick are nested and the intersection of these cones forms a chain of causal areas (Fig. 1C). We call the original events ‘audible’ and the audible event sequence of our clock is the discrete analog of a particle’s worldline. The corners in the chain of areas that are not audible we call ‘inaudible’. They are not part of the event sequence that is the analog of the worldline but their positions in spacetime help determine the invariant areas between ticks. Fig.[1D] shows that the chain of areas inherit an alternating orientation from a single path that joins all original events at path crossing points.

Figure 1: A periodic sequence of events has an associated sequence of forward and reverse light cones (B). The intersection of the events and the light cone area between the events sets up a chain of ‘causal’ areas (C). If the original events are the intersections of locally smooth curves, the areas are oriented (D).

The clock of Fig.[1] ticks at the integers in the rest frame of the clock. To interpolate between events we need to use the chain of oriented areas to define a discrete function that may then be used at higher frequencies to return a smooth interpolation. To this end consider Fig.[2]. We use the right-hand boundary of the oriented areas to map the orientation onto a two-component column vector. The resulting clock then has four states. The clock, or state variable $s_k$, is a characteristic function for orientation and it takes on one of four values:

$$s_k \in S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

in succession. If the clock is in state $s_k$ at time $k$ then the subsequent state after the next event is

$$s_{k+1} = T s_k$$

where $T$ is the transfer matrix:
Since $s_k = T^k s_0$, the power of the transfer matrix corresponds to the (discrete) displacement in time.

To obtain continuous time we need a clock of arbitrarily high frequency to allow for real values of $t$. If $T_\nu$ is the transfer matrix for a high-frequency N-clock, synchronized with $T$, with intrinsic frequency N times that of the original (Fig.[3]), we must have an equivalence between the N-th power of $T_\nu$ and $T$ itself. Thus

$$T_\nu^N = T$$

(4)

giving $T_\nu$ as an Nth root of T or

$$T_\nu = \begin{pmatrix}
\cos \left( \frac{\pi}{2N} \right) & -\sin \left( \frac{\pi}{2N} \right) \\
\sin \left( \frac{\pi}{2N} \right) & \cos \left( \frac{\pi}{2N} \right)
\end{pmatrix}$$

(5)

Figure 2: We use the right boundary of the chain of oriented areas to construct a two component state vector. The upper component registers a $\pm 1$ or 0 for the signed projection onto the right light cone. The lower component registers the left light cone.

Taking the limit as $N \to \infty$ via an eigenvalue expansion we get the transfer matrix

$$T_R(t) = \lim_{N \to \infty} T^{Nt}_\nu = \begin{pmatrix}
\cos \left( \frac{\pi t}{2} \right) & -\sin \left( \frac{\pi t}{2} \right) \\
\sin \left( \frac{\pi t}{2} \right) & \cos \left( \frac{\pi t}{2} \right)
\end{pmatrix}$$

$$= I_2 \cos \left( \frac{\pi t}{2} \right) + T \sin \left( \frac{\pi t}{2} \right)$$

(6)

The dependence of TR on real values of $t$ allows us to have access to analog clocks in the rest frame of our original digital clock. Notice that TR reproduces the events of the digital clock at the even integers but interpolates between the original discrete states via a rotation. To allow for moving frames we have to adjust the transfer matrix to permit the two different projections onto the left and right light cones Fig.[3 B]. The result is [9]
Here \( v \) is the velocity of the moving frame and \( \gamma = \frac{1}{\sqrt{1-v^2}} \) is the time dilation factor normalizing the ‘vector’ \( M = \gamma(v\sigma_z + i\sigma_y) \) that constitutes the kinematic part of the clock. Notice that \( M^2 = -I_z \) and \( M \) is the replacement of the tick matrix \( T \) for the moving clock. The coefficient of \( M \) contains the dynamic phase and frequency information of the clock. We call the clock associated with (7) a Compton clock for reasons that shall appear shortly.

\[
T_M(t) = \begin{pmatrix}
\cos\left(\frac{\pi t}{2\gamma}\right) - v\gamma \sin\left(\frac{\pi t}{2\gamma}\right) \\
\gamma \sin\left(\frac{\pi t}{2\gamma}\right) \\
\cos\left(\frac{\pi t}{2\gamma}\right) + v\gamma \sin\left(\frac{\pi t}{2\gamma}\right)
\end{pmatrix}
\]

\[
= \cos\left(\frac{\pi t}{2\gamma}\right) I_2 - \sin\left(\frac{\pi t}{2\gamma}\right) \gamma(v\sigma_z + i\sigma_y)
\]

(7)

Figure 3: Contact with continuous time is made by allowing arbitrarily high frequency Compton clocks to fill in events on the \( t \) axis. Note how the area between events decreases as events are filled in. As the event frequency increases the causal area between events goes to zero.

It is useful to take stock at this point and compare eqn(6) with eqn(7). Newtonian mechanics allows an absolute time that is independent of the motion of the observer. In the case of absolute time, eqn(6) could be used as the reference clock for all inertial frames and since \( T \) is a representation of the unit imaginary, our reference frame clock would effectively be a translationally invariant unimodular complex number. There would then be little need to consider the clock aspect of objects since the argument of \( TR \) would not depend on relative speeds and could be reasonably assumed to be ‘available’ to all objects. This would seem to justify Newton’s absolute time and the segregation of time from space as a separate and independent coordinate.

In contrast, we see from (7) that with the relativity of simultaneity, space and time are intrinsically linked. The linkage here is not through the invocation of an ambient spacetime but simply through the geometry of a sequence of clocks. In this view, absolute time is only an approximation and instead of a simple unimodular complex number appropriate for all frames we see the implication of geometric algebra [10] that allows for moving frames. Noting that \( M^2 = -I_z \) we see the origin of the odd signature of spacetime from the fact that spacetime areas in the plane defined by the \( t \) and \( v \) axes are invariant. The necessity to go beyond the single analog clock of Newton to require the Dirac algebra for a four dimensional spacetime is a direct consequence of the differing frequencies of clocks on the two lightcones in the reference frame moving with a non-zero relative velocity Fig.[4(b)]. The full
Dirac algebra is required in three dimensions of space since we need two more ‘vectors’ in the geometric algebra sense to embed our analog clock in three space.

Figure 4: (a) A Compton clock at rest provides a chain of spacetime areas. To interpolate between events we construct higher frequency Compton clocks giving the continuous transfer matrix TR. (b) In a moving reference frame the regions between events become rectangular, however the area is preserved and is calculated as the product of the lengths of the two sides of the rectangles. The effect on the transfer matrix is to weight the residence times of the two states differently.

One more feature is worth noting at this point. To arrive at eqn(7) we had to take an infinite frequency limit of the Compton clock. It is this limit that establishes the Lorentz covariance of the spacetime frame that we are building from our digital clock. However, from the perspective of the clock, the reference frame is an infinite frequency idealization. If classical ‘particles’ are simply small clocks that obey special relativity through geometry, the association of a smooth worldline with their passage through time will have the same effect. The arbitrarily high frequency of events implied by the association of ‘event’ with spacetime position on a smooth worldline is an infinite frequency construction that, from the perspective of quantum mechanics, means infinite mass! This suggests that to get to the kinematics of conventional special relativity, we have ‘frozen out’ quantum propagation with an infinite mass limit. To introduce dynamics into relativity, mass has then to be reinserted as a background feature. Our clock has implicated the metric features of a Clifford algebra of odd signature through the ‘vector’ M by ignoring the dynamic part of the signal (the trigonometric functions) in the transition to spacetime. If the spacetime algebra is all we extract from the clock, we remove the connection to signal processing and the uncertainty principle.

It seems clear from the above that to extract spacetime we have taken an infinite frequency limit that could be expected to eliminate quantum propagation via the quantum Zeno effect. This suggests that we ask if it is possible that quantum propagation is implicated if we forgo the infinite frequency limit underlying (7)?

Consider Fig.[5]. The digital clock with the right hand path used for counting appears in the first two frames. The third frame is a sketch of a path in which the distance between the corners is stochastic. If the distribution governing the distance between corners is exponential and the matrix T governs the corners of the paths, the resulting model is the Feynman chessboard model. This is Feynman’s sum-over-paths formulation for the Dirac propagator. [11-15].

The appearance of Dirac propagation at this point may appear paradoxical. We have shown that classical spacetime emerges as a high frequency limit of the digital clock. The clock construction itself is a simple algorithm for counting and there has been no quantization procedure that would necessitate
interpretation after the calculation. Mass does not appear as a characteristic frequency in the digital clock since the high frequency limit leaves only the scale of the unit vectors of the spacetime frame. How then is wave propagation implicated in the chessboard model when there is none in the limit of Compton's clock?

Figure 5: (A) Compton's clock. (B) The right hand boundary used to form the characteristic function for counting (Fig. 2). (C) A stochastic version of (B) used in the Feynman chessboard model. The orientation of the areas in (A) provide the 'corner rule' for Feynman's model.

From Fig.[5C] we see that the difference between Compton's clock and the chessboard model is the stochastic element built in to the causal areas. The Poisson process that governs the limit allows arbitrarily high frequency but gives a finite expected frequency at the Compton frequency. The remnant of Compton's clock is the 'zitterbewegung' that appears in the Dirac equation [16].

Notice that the chessboard limit preserves the indeterminacy feature of the original digital clock. The rectangular area between two audible events in the Compton clock is causal in the sense that the clock mechanism is confined to that area. In the continuum limit for the Compton clock, the causal area goes to zero as the event density increases. In the chessboard model, the continuum limit does not reduce the causal area because it does not confine the ticking of the clock to the linear interpolant between any two events; that is the classical spacetime limit. Instead, the chessboard limit allows the clock to tick any integral number of times between the initial and final tick. The implementation of this allows the enumerative paths of a stochastic clock (the Chessboard paths) to cover the full causal area between two audible events.

Figure 6: At fixed velocity, the length scale at which the moving clock has lost one tick is the deBrogli scale.
The Dirac propagator implicated by the Feynman paths explicitly shows that there are two characteristic lengths arising from this picture of particle-as-clock. The inner scale is determined by the mass of the particle, the Compton length. The outer scale is determined by the initial and final events in combination with boundary conditions if the spatial domain is restricted. This is not however how conventional non-relativistic quantum mechanics sees quantum propagation. The Compton scale does not arise in the Schrodinger equation and we might question how the wave propagation we see here, so obviously relativistic in origin, could scale up to the deBroglie length.

The answer is remarkably simple. 'Moving clocks run slow' in special relativity and the length scale at which a moving clock resynchronizes with a stationary clock, after missing a single tick, is the deBroglie wavelength (Fig. 6). Using an analogy with sound, the deBroglie wavelength is simply the beat frequency arising from the fact that the Compton frequency of the moving clock is slightly lower than the frequency of a stationary clock due to time dilation. If the geometry of any confinement of the clock is closer in scale to the deBroglie length than the Compton length, (as it is for example in the case of the electron orbitals of the Hydrogen atom) the result is that the deBroglie length and period become 'fundamental' to the clock. This suggests why the Bohr-Sommerfeld rules provided such a surprisingly accurate account of the spectral lines of Hydrogen prior to the advent of modern quantum theory. The quantized angular momentum rules are a generalization of the synchronization of stationary and moving clocks that lose an integral number of ticks in an orbit [9, 17].

3. Conclusions

A frequent starting point for an invocation of Minkowski space is to note that for a light wavefront

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = 0.$$ (8)
regardless of the inertial frame of reference. If we start with (8) we can quickly invoke spacetime algebra [10] and then bring in mass through dynamics.

From the perspective of Compton's clock, we can see where the odd signature of this starting point comes from, but we also see that by just invoking an appropriate algebra, we miss the dynamic aspect of the clock signal. Equation(7) shows that spacetime captures the kinematics that are features of all clocks, regardless of their fundamental frequency and phase. The dynamical signal along with its association with signal processing and the uncertainty principle is however lost.

The Feynman chessboard model keeps the dynamic information and does not enforce a smooth worldline. The result is a 'clock' that satisfies the Dirac equation. The difference between Compton's clock and the chessboard model is how the continuum limit is taken. As illustrated in Fig.(7) the Compton clock enforces a linear worldline by taking an infinite frequency limit between all ticks of the original digital clock. The chessboard model allows arbitrarily high frequency but weights inter-arrival time with an exponential distribution, thereby preserving an aspect of wave propagation, including the uncertainty principle. As models of clocks, the Compton clock and the chessboard model differ only in the choice of constants and the method of taking continuum limits, the latter postponing the continuum limit to values of t greater than the Compton length in the system. Since Compton's clock is just a simple classical relativistic clock in which the origin of phase is time dilation, we see that Dirac propagation and hence quantum mechanics are ultimately manifestations of special relativity in which the continuum limit preserves the uncertainty principle.

References

On the shoulder of giants

\[ E_{QR} = (\frac{\phi^5}{2})(mc^2) \approx mc^2 / 22 \]
\[ E(D) = \left(\frac{5\phi^2}{2}\right)mc^2 \approx mc^2 \left(\frac{21}{22}\right) \]

On the Shoulder of Giants

Galileo Galilei  Isaac Newton  Georg Cantor

Henri Poincaré  George Fitzgerald  Hendrik Lorentz

Albert Einstein  Max Planck  Max Born
The above photo gallery is a non-exhaustive collection of scientists, colleagues and teachers whose work and influence helped in discovering $E(0) = mc^2 / 22$ and $E(Dark) = mc^2 (21/22)$ where $E(0)$ is the ordinary energy of the quantum particle which we can measure and $E(Dark)$ is the energy of the quantum wave which we cannot measure.